Factor Structure in Cryptocurrency Return and Volatility*

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First draft: 05/16/2019

Current draft: 06/14/2021

Abstract

We use high-frequency tick data to study stylized facts on the return and volatility dynamics of the nine most liquid cryptocurrencies. Factor structures exist in both returns and volatility, but the explanatory power from the common factor is much stronger for volatility. The factor structures do not relate strongly to fundamental economic factors, and Bitcoin – which we propose is a "crypto market factor" – has only weak explanatory power. We date the bubble in Bitcoin pricing allowing us to split the sample into pre-bubble, bubble and post-bubble periods. The importance of these different periods is clear, revealing shifting relationships between the nine cryptocurrencies and Bitcoin. Model-free realized cryptocurrency betas with Bitcoin increase during the bubble and the explained fraction of cryptocurrency variance remains at an elevated level after the bubble burst.

JEL Classification: C38; G12; G13

Keywords: Cryptocurreny; Factor Structure; Realized Volatility; Bitcoin; Market Bubble

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*We thank for participants at Cass PhD Research Day, 3rd Edinburgh-SUFE Shanghai Fintech Conference, ESSEC WORKSHOP on Nonstandard Investment Choice and Southwestern Financial Association 2021 for helpful comments. All errors are our own.

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1 Introduction

Cryptocurrencies have caught the eye of individual and institutional investors, primarily because of the exceptional returns they have offered. Though they have been in existence since 2008, the year Bitcoin was invented by Nakamoto,[1] the most critical period in the history of cryptocurrencies is the so-called Bitcoin bubble. Between April 2017 and December 2017, the dollar price of Bitcoin rose from $600 to $19,815. On December 16, 2017, as the Bitcoin price reached a historical high, *The Wall Street Journal* published an article entitled "Is Bitcoin a Bubble? 96% of Economists Say 'Yes'". From January to February 2018, the Bitcoin price fell by 65%.

Despite this enormous Bitcoin price fall, which was shared by many cryptocurrencies, the total crypto market capitalization remains substantial; in September 2018, it was around $208 billion. As cryptocurrency trading has become more popular, finance academics have been drawn to examine the market, starting with Yermack (2015). Many subsequent studies consider cryptocurrencies from an asset pricing perspective (Weber, 2016; Chiu and Koeppl, 2017; Abadi and Brunnermeier, 2018; Huberman et al., 2019; Cong and He, 2019; Schilling and Uhlig, 2019; Cong et al., 2019; Biais et al., 2019; Cong et al., 2020; Sockin and Xiong, 2020; Saleh, 2020).

Despite this increased interest, many characteristics of cryptocurrencies as financial assets remain unclear. Our main objective in this study is to document the factor structures of returns and volatilities in the nine most liquid cryptocurrencies quoted against Bitcoin between October 2016 and November 2018 using high-frequency quote and transactions data. We go on to test whether the common components of returns and volatilities are driven by major macroeconomic factors, and how the crypto factor structures were affected by the Bitcoin pricing bubble. Finally, we test whether Bitcoin acts as a fundamental market factor in the cryptocurrency market.

We demonstrate nine stylized facts:

Fact 1: *Daily realized cryptocurrency volatility has high persistence.*
Fact 2: *The distribution of the logarithm of realized volatility of cryptocurrencies is close to normal.*
Fact 3: *The factor structure in daily cryptocurrency volatility is stronger than the factor structure in returns.*
Fact 4: *Economic and financial factors do not have strong explanatory power on the common factors of cryptocurrency return and volatility and there is a weak inverse relationship between cryptocurrency risk and macroeconomic indices.*
Fact 5: *Bitcoin can be considered for most cryptocurrencies as a fundamental factor able to*

[1] See the study by Nakamoto (2019)

explain a small proportion of the variations in return and volatility.
Fact 6: *The Factor Structure model is more powerful in explaining variation in returns and volatilities during the Bitcoin bubble period and this explanatory power persists - and for volatilities actually increases further - after the Bitcoin bubble burst.*
Fact 7: *There is heterogeneity in the relationship between Bitcoin and other cryptocurrencies for both returns and volatility after the Bitcoin pricing bubble burst.*
Fact 8: *Cryptocurrency betas with Bitcoin were negative before the Bitcoin bubble burst but became positive after the bubble burst.*
Fact 9: *The fraction of variance of cryptocurrency explained by the Bitcoin variance is high during the bubble period, and the explained fraction remains at an elevated level in the post-bubble period.*

Bianchi (2020) conducts an empirical study on the returns of cryptocurrencies and traditional financial assets. His main finding is that there is no significant correlation between cryptocurrency returns and the return of traditional financial assets. Only gold and crude oil have weak correlations with cryptocurrency. A recent study by Liu and Tsyvinski (2018) finds similar results, concluding that cryptocurrency prices contain no information related to other financial assets or pricing factors. However, Liu and Tsyvinski explore the theory that the returns of cryptocurrency are predicted by factors that are specific to cryptocurrencies. This may imply that the information explaining cryptocurrency is not shared with traditional financial assets in the financial market. In other words, questions on the cryptocurrency market should be investigated by focusing on the inherent characteristics belonging to cryptocurrency instead of naively borrowing from studies on traditional financial assets.

The remainder of the chapter is structured as follows. In section 2, we describe our data and methodology for computing returns and estimating realized volatility. We construct factor structure models in section 3. The explanatory powers of economic factors on the common components in cryptocurrencies are tested in section 4. Section 5 detects the timing of the Bitcoin price bubble and the impact of the bubble on the factor structure of other cryptocurrencies. In section 6 we estimate cryptocurrency market betas and compute systematic risk ratios contributed by Bitcoin. We draw our conclusions in section 7. Supplementary figures and tables can be found in the Appendix B.

2 Construction of Returns and Realized Volatility

2.1 Cryptocurrency Data

We obtain intraday trading data on cryptocurrency from Kaiko, a company that collects tick data pertaining to cryptocurrencies. Kaiko provides tick by tick data on more than 200 cryptocurrencies traded on 15 large and liquid cryptocurrency exchanges.[2] As explained below, we augment the Kaiko data with similar data from CoinAPI.io, a company that provides a similar service to Kaiko, providing cryptocurrency data accessed through querying APIs from multiple exchanges.

We make several methodological decisions regarding use of the source data. First, we analyse cryptocurrency exchange rates against Bitcoin (Cryptos/BTC) rather than crypto rates against fiat currencies such as the U.S. dollar. The extreme price changes of Bitcoin versus the dollar noted above were mirrored by most other cryptocurrencies. Studying crypto exchange rates against the dollar would have inevitably uncovered enormous common structures, as cryptos first rose and then fell against the dollar - or any other non-crypto base price. While this is an important issue to consider it is not what we wished to examine in this study. Instead, we focus on testing for common structures *between* cryptocurrencies and instead use the BTC/USD boom and bust episodes as sub-periods for our tests. Since Bitcoin is the headline cryptocurrency, we use it as the base price against which all crypto exchange rates are measured.

Second, and following from the decision to focus on BTC-cross crypto rates, we take data from the Bittrex exchange, a leading exchange located in Seattle that mainly facilitates trades of cryptos against Bitcoin. Makarov and Schoar (2020) have noted that cryptos often trade at markedly different prices on different exchanges; hence to ensure comparability it is important that all rates come from the same exchange.[3]

Finally, though many cryptos are traded at the same time, many do not survive long, and many others have only recently been introduced. We select nine cryptocurrencies that have had data available throughout the sample period from October 2016 to November 2018. These nine currencies are Ethereum (ETH), Ethereum Classic (ETC), Ripple (XRP), Litecoin (LTC), Dash (DASH), Zcash (ZEC), Lisk (LSK), Monero (XMR), Stratis (STRAT). There is a clear and conscious selection bias inherent in this decision. Our results pertain only to this set of relatively long-lived cryptocurrencies selected for the very reason that they have survived.

[2]The exchanges are Bitstamp, Kraken, BTCC, Bittrex, Coinbase, OkCoin, Bitfinex, Poloniex, Bithumb, Gemini, Quoine, bitFlyer, Huobi, Binance and Zaif.

[3]Several other papers document potential problems of investment in cryptocurrencies, including Borri and Shakhnov (2018), Hu et al. (2019) and Borri (2019).

Table 1: Summary of Cryptocurrency

The table shows all cryptocurrency used in our study. For each asset, we report the trading symbol time-zone, market capitalization, close price, circulating supply, and percentage of total market capitalization in the cryptocurrency market. The summary data is from https://coinmarketcap.com. All statistical data is up to November 2018 which is the last month in our sample period.

Currency	Symbol	Time Zone	Market Cap	Price	Circulating Supply	% Total Market Cap
Bitcoin	BTC	UTC	$65,549,846,077.00	$3,768.79	17392787	54.28%
Ripple	XRP	UTC	$13,998,356,446.00	$0.35	40327341704	11.59%
Etherrum	ETH	UTC	$11,158,159,719.00	$107.90	1866712302	9.24%
Litecoin	LTC	UTC	$1,692,307,423.00	$28.54	59229875	1.40%
Monero	XMR	UTC	$929,735,016.00	$56.02	16596133	0.77%
Dash	DASH	UTC	$743,512,468.00	$87.85	8463191	0.62%
Ethereum Classic	ETC	UTC	$478,701,141.00	$4.50	106284797	0.40%
Zcash	ZEC	UTC	$339,981,605.00	$64.03	5309689	0.28%
Lisk	LSK	UTC	$146,100,728.00	$1.30	112501790	0.12%
Stratis	STRAT	UTC	$64,322,236.00	$0.06	99106480	0.05%

Table 1 summarizes the cryptocurrencies' overall market capitalization, volume and circulating supply at the end of November 2018. Including Bitcoin, the cryptocurrencies we study in this chapter represent almost 79% of the total market capitalization of the cryptocurrency market.

2.2 Other Data

We collect commodity futures and foreign exchange spot data from Thomson Reuters Tick History (TRTH) at the minute frequency. Specifically, we use commodity futures on crude oil, gold, S&P500 E-mini, and CBOE SPX VIX, and foreign exchange spot data for CNY/USD and EUR/USD. Our cryptocurrency study focuses on the interval between October 2016 and November 2018 and makes use of this market's 24-7 continuous trading feature. Analysis using the foreign exchange factor is from February 2017 to November 2018 since offshore trading in the Chinese currency started in February 2017, as discussed further in section 4.

We follow the standard high-frequency data cleaning process to remove bad data points. To be exact, we follow the first three steps of the quote data cleaning processes described by Barndorff-Nielsen et al. (2009). We do not conduct the fourth step that eliminates extreme quotes because we want to preserve the nature of cryptocurrency trading as much as possible. Nevertheless, the results of our analysis are insensitive to the removal of extreme quotes.

All cryptocurrency and financial products' daily realized volatilities are calculated from minute-sampled mid-quote data after the data cleaning procedures.

2.3 Return and Realized Volatility Calculations and Data Cleaning

We analyze daily return and realized volatility measures for our set of nine cryptocurrencies. In theory, given that the crypto market trades continuously over seven days per week, calculating these measures should be straightforward. Unfortunately, the data are imperfect and there are intervals where relevant observations are missing. We first explain the methods used to calculate our key measures on the assumption of perfect data and then detail how we deal with the missing data.

We construct realized volatility following Christoffersen et al. (2019) and Zhang et al. (2005). Each day has an $(n+1)$ 1-minute time-grid price. The n 1-minute time-grid returns at day t are calculated as:

$$r_{t_j} = log(Mid_{t_j}) - log(Mid_{t_{j-1}}) \tag{1}$$

where $t_j - t_{j-1}$ is equal to one minute and $log(Mid_{t_j})$ is the mid quote of logarithm of ask price and logarithm of bid price. We then calculate each five-minute return by summing the five one-minute returns:

$$\widetilde{r}_{t_k} = \sum_{k=j}^{j+4} r_{t_k} \tag{2}$$

Each day will have $(n-4)$ five-minute return. Finally, the daily measure of 5-minute realized volatility calculated with 1-minute subsampling is defined as :

$$RV_t^{oc} = \frac{n}{5(n-4)} \sum_{k=1}^{n-4} (\widetilde{r}_{t_k})^2 \tag{3}$$

Using subsampling techniques to calculate 5-minute returns reduces market microstructure noise in the volatility estimate.[4]

The Kaiko data provide minute snapshots of the crypto orderbook up to ten levels from the best bid and ask prices (giving both price and depth data). In theory, since cryptos trade around the clock, seven days per week, we should observe 1440 snapshots of the data throughout our sample. Unfortunately, this is not the case. We therefore check whether the missing prices can be filled in with data from the CoinAPI.io database. In theory, this should be a reasonable solution since when both Kaiko and CoinAPI provide data for the same crypto from the same exchange, the data are exactly comparable. Nevertheless, even after filling in all possible missing observations, data are still sometimes missing, particularly in the April-August 2017 interval. That data are missing in this interval is probably not random. The Bitcoin price was rising rapidly at this time and

[4]The market microstructure noise issue on high frequency data has been well discussed by Campbell et al. (1997), Andersen et al. (2005) and Ait-Sahalia et al. (2005).

trading was extremely active. We suspect that data providers struggled to keep up with orderbook developments leading to data problems.

As a result of this problem, we encounter some days with intervals during which no orderbook data are available. We adopt two methods to solve this issue. Our first approach is to follow Müller et al. (1990) Dacorogna et al. (1993) and Andersen et al. (2001). This involves simply interpolating in a linear fashion across intervals in the data as long as the interval is small enough for this to be reliable. To decide what constitutes a small enough interval we run the following test.

For each currency, we extract those days with the full 1440 minutes of data. We randomly delete observations within the day creating missing data intervals of length j-minutes. These intervals are then re-filled by linearly interpolating across the gap. We then calculate the daily realized volatility as discussed below. One realized volatility is calculated for the original full data set and the other is calculated using the data set containing j minutes of interpolated data. Finally, we compute the correlation between the two computed realized volatilities. We deem the interpolation to be acceptable if the correlation is greater than or equal to 0.98. In practice, we conclude that data can be linearly interpolated up to $j = 200$ minutes without loss of accuracy. Above that, the correlation is unacceptably low and a second econometric method has to be employed.

We also test the power of linear interpolation on days with multiple missing data intervals (for example, we may have ten missing intervals in the data during a day, each 50 minutes long but separated in each case by ten minutes of observed data). Perhaps unsurprisingly, the problem of several relatively small gaps in a day is far less severe than the problem arising from one long missing interval. The example of one day with ten 50-minute intervals is acceptably corrected by using linear interpolation across each gap in the data, even though the intervals total some 500 minutes. This is much longer than the single interval that can be successfully interpolated. In summary, as long as the data missing between two timestamps do not exceed 200 minutes, we use linear interpolation.[5]

For intervals longer than 200 minutes, we use a second procedure in line with that used by Hansen and Lunde (2005). Their method is designed to account for systematic breaks in trading as is typically observed in stock markets. Hansen and Lunde (2005) propose that both the realized variance computed from high-frequency data during trading hours and the squared close-to-open return (r^{co}) over an inactive period contain information relevant to computing the integrated variance (IV) of an asset.[6]

[5] We also check whether other methods such as Spline or Lagrange interpolation perform better than linear interpolation. The results are very similar to linear interpolation.

[6] See the detailed analysis by Andersen and Bollerslev (1998).

To minimize the difference between realized variance and integrated variance,[7] Hansen and Lunde (2005) develop optimal weights for r_t^{co} and RV^{oc}, which remove much of the noise due to using high-frequency data:

$$RV_t(\omega) = \omega_1 (r_t^{co})^2 + \omega_2 RV_t^{oc} \tag{4}$$

The Hansen and Lunde (2005) technique can be applied easily if trading breaks are of equal duration and occur each trading day, since the parameters driving the optimal weights can be estimated from simple sample averages. However, in our cryptocurrency data, the trading breaks occur at different points during the day, are of different lengths and only occur sporadically. We therefore adapt the Hansen and Lunde (2005) approach accordingly as follows.

For days with a single trading break (longer than 200 minutes) on day t, we simulate all other days with full data availability to have the exact same trading break (occurring at the same time, and for the same interval). We then apply the Hansen and Lunde (2005) technique outlined above (and described in further detail in their paper) to calculate the optimal weights for the close to open squared return and the open to close realized volatility.

For days having more than two breaks we adapt the Hansen and Lunde (2005) method and apply:

$$RV_t(w) = \hat{w}_1 \sum_{i=1}^{B} (r_{i,t}^{co})^2 + \hat{w}_2 \sum_{i=1}^{B+1} RV_{i,t}^{oc} \tag{5}$$

where B is the number of breaks, $RV_{i,t}^{oc}$ is the realized volatility between each break calculated from equation (3), and $r_{i,t}^{oc}$ is close to open returns between breaks. We again create simulated data with exactly matching breaks from those days with complete data and proceed as usual.

The combination of simple linear interpolation across small gaps in the data and the Hansen and Lunde weighting when there are longer gaps allows us to compute daily realized volatilities for all currencies. To compute daily returns, as used in the analysis below, we need a price at midnight each day. On some days, there are trading gaps spanning midnight. We therefore linearly interpolate between the last available mid-price on day t and the first available price on day $t+1$ to obtain the midnight price.

2.4 Properties of the Cryptocurrency Daily Returns and Volatilities

Table 2 provides descriptive statistics of the daily log returns of the nine cryptocurrencies. All nine exhibit positive skewness, and the extreme values - both maxima and minima - are dramatic. The first order autocorrelation does not show strong persistence at the 1% level except for ZEC and

[7] That is, $\min_{\omega} E[RV_t(\omega) - IV_t]^2 = 0$

Table 2: Sample Statistics of Cryptocurrency Return

The table shows sample statistics for daily log return for all cryptocurrency during the October 2016-November 2018 period. ACF(1) denotes the first-order autocorrelation. Q(L) is the Ljung-Box test of zero autocorrelation in lags 1 through L. An asterisk indicated at the 1% level.

Statistics	ETH	XRP	STRAT	LTC	ETC	DASH	ZEC	LSK	XMR	BTC
Mean	0.03	0.25	0.16	0.03	-0.07	0.02	-0.61	-0.03	0.02	0.23
Std.	5.22	8.76	7.68	5.44	6.25	5.69	7.23	7.26	5.59	4.42
Min	-25.88	-68.49	-27.11	-23.12	-28.38	-28.85	-51.78	-38.00	-26.36	-17.14
25%	-2.25	-2.87	-3.98	-2.04	-2.89	-2.61	-3.30	-3.67	-2.62	-1.45
50%	-0.29	-0.55	-0.62	-0.46	-0.59	-0.37	-0.79	-0.72	-0.36	0.37
75%	1.52	1.78	3.40	1.30	1.70	1.96	1.74	2.66	2.08	2.31
Max	32.78	101.20	48.91	56.32	54.23	46.46	46.96	40.69	36.96	23.82
Skewness	1.21	2.85	1.07	2.80	1.43	1.27	0.02	0.75	1.24	-0.04
Kurtosis	10.04	36.38	7.83	24.78	13.58	13.84	13.44	8.18	10.67	6.28
ACF(1)	0.07	-0.02	-0.004	0.02	-0.0002	0.04	0.18*	0.09*	-0.06	0.01
Q(5)	16.06*	12.07	6.58	6.20	13.74	2.02	51.19*	15.97*	11.31	8.35
Q(21)	43.55*	44.68*	55.29*	34.44	52.79*	30.57	77.07*	38.58	27.96	24.97

LSK, and the Ljung-Box test shows no significant persistence across 5 to 21 lags. Figure 1 plots the autocorrelation function up to 60 lags confirming that cryptocurrency daily returns do not show high persistence.[8]

Figure 2 plots the daily realized volatility (RV_t) of the nine cryptocurrencies and Panel A in Table 3 reports descriptive statistics. As expected, the cryptocurrencies are very volatile and the RVs of all cryptocurrencies also have high positive skewness and kurtosis. The maximum daily volatility in our sample period is extremely large, even compared to commodities (see the study by Christoffersen et al. (2019)). More importantly, the first-order autocorrelation is large and significant at the 1% level for all cryptocurrencies, and the Ljung-Box test statistics are also strongly significant across both 5 and 21 lags. It is clear that the realized volatilities of cryptocurrencies are highly persistent and this is the first stylized fact we report:

Fact 1: *Daily realized cryptocurrency volatility has high persistence.*

Panel B in Table 3 reports sample statistics of the natural logarithm of realized volatilities. This does not alter our conclusions regarding the persistence of realized volatilities (see Figure 3) but the log transformation changes the data distribution dramatically (see Figure 4). The skewness of log realized volatilities are all close to zero, much reduced from levels reported in Panel A. All log realized volatilities have kurtosis levels close to three. Figure 5 gives the QQ plot of $log(RV_t)$ for the cryptocurrencies, demonstrating the near normality of log realized volatilities. The effects of taking logarithms of realized volatility have been addressed for equities by Andersen et al. (2001), for the foreign exchange market by Andersen et al. (2001) and in commodity markets by Christoffersen et al. (2019). To the best of our knowledge, we are the first to document that:

[8] The price level and return figures of the nine cryptocurrencies can be found in Appendix B

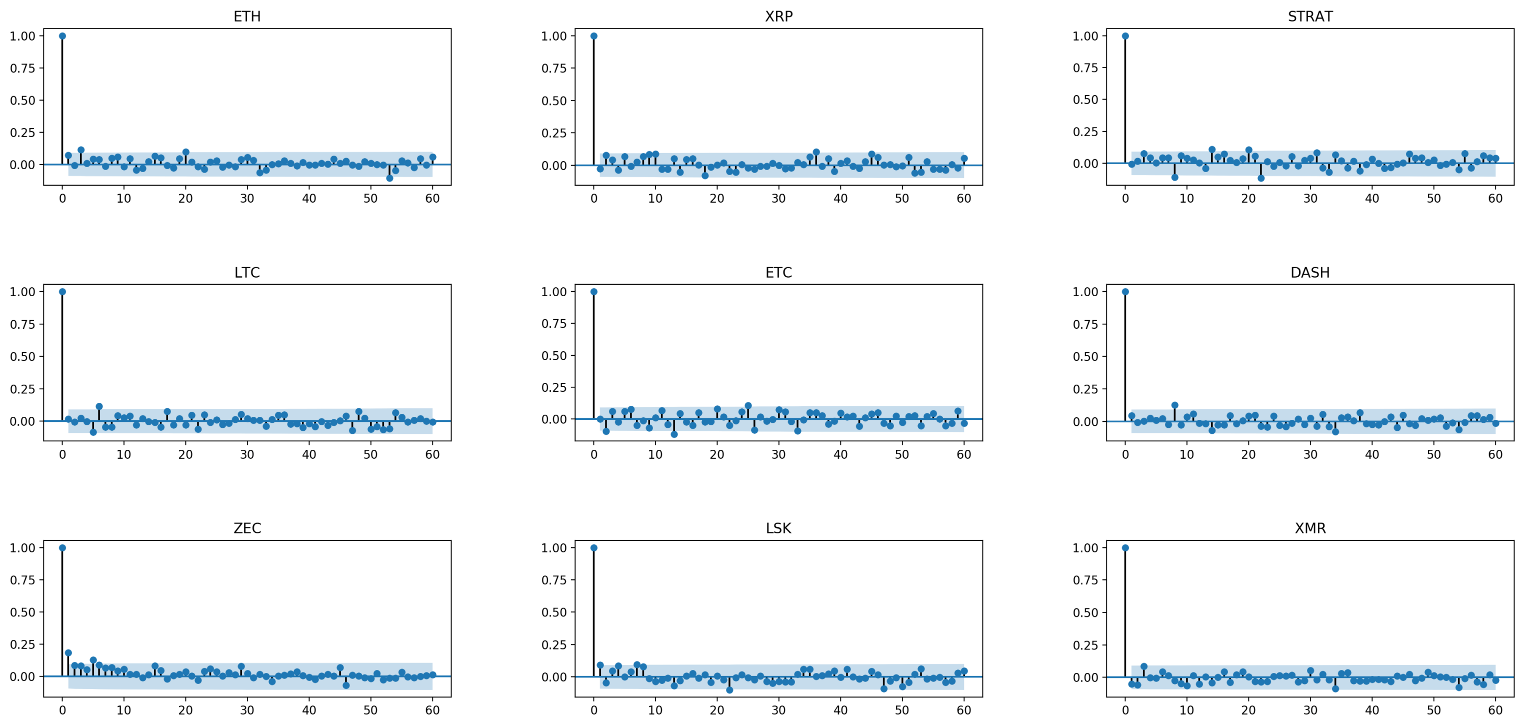

Figure 1: Cryptocurrency Daily Return Autocorrelation

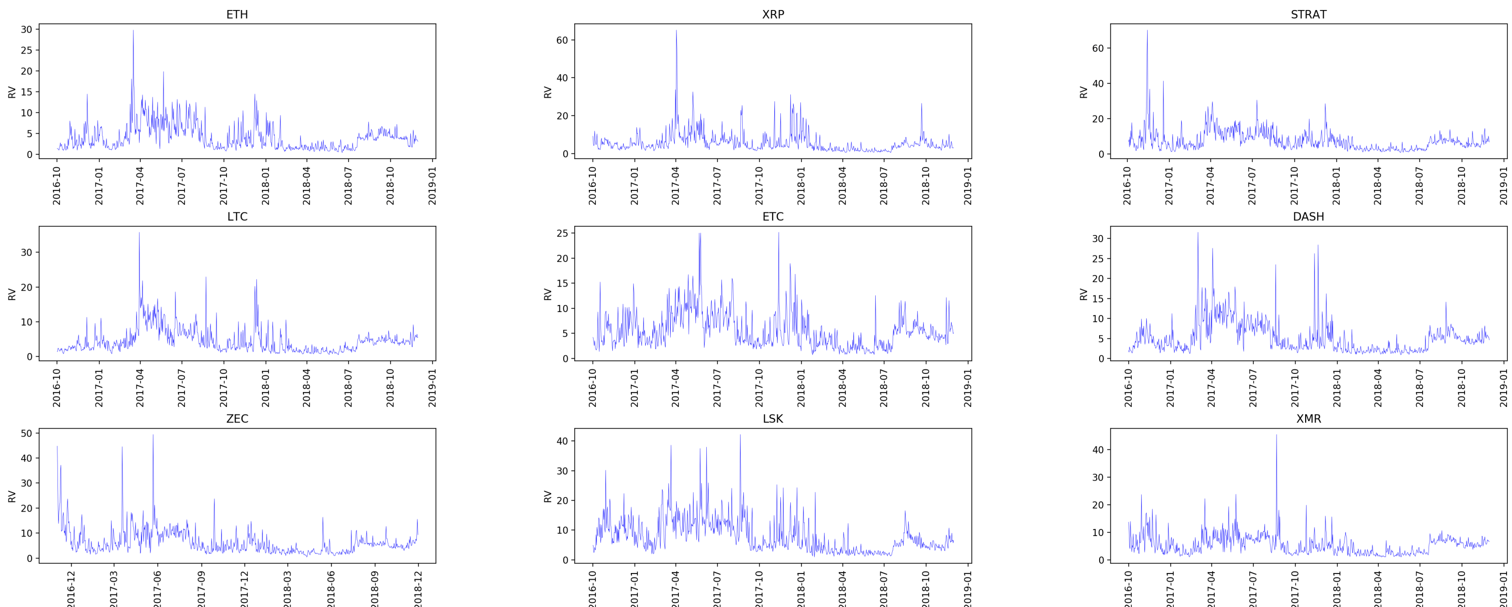

Figure 2: Cryptocurrency Daily Realized Volatility

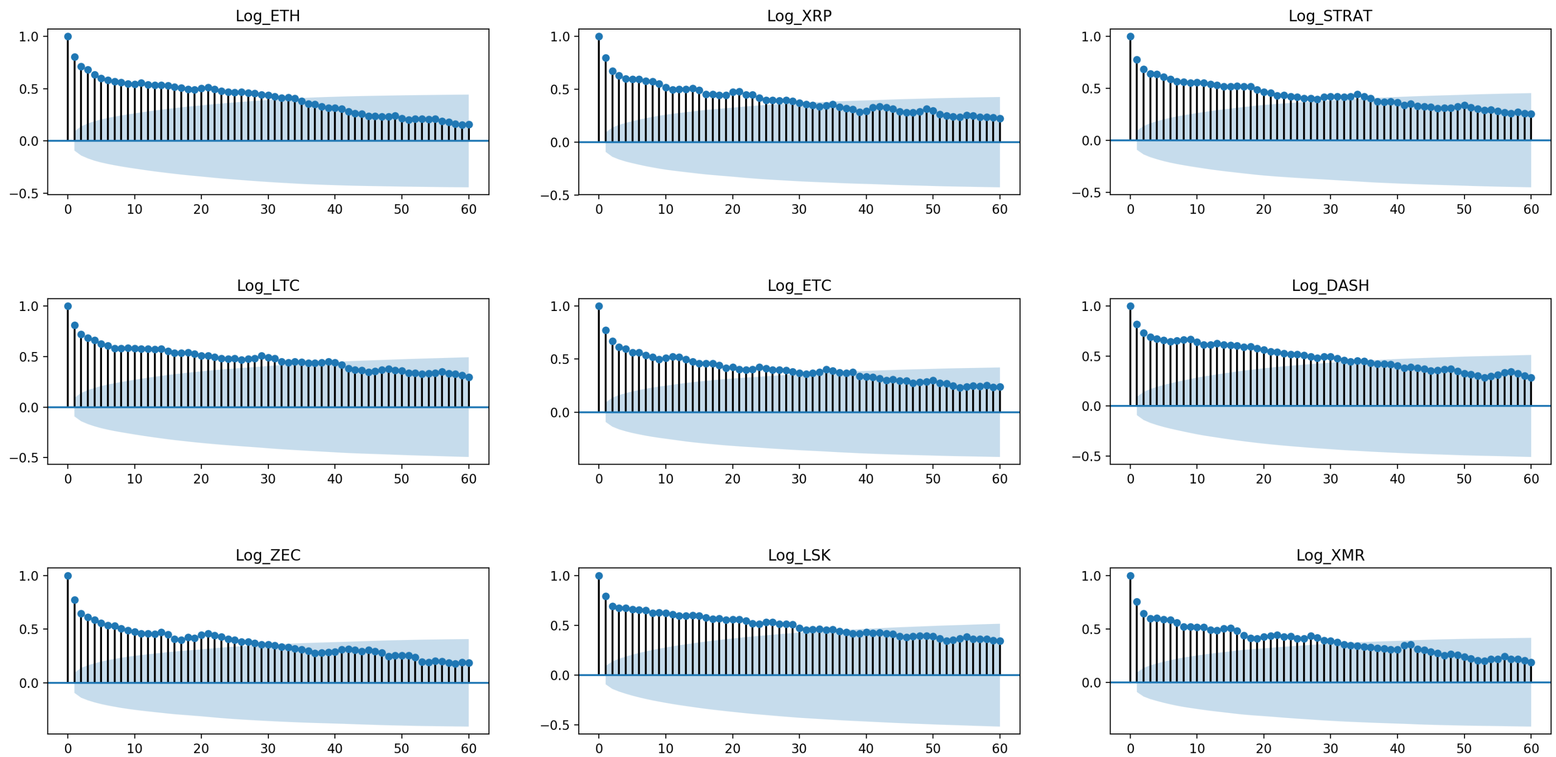

Figure 3: Log RV_t Autocorrelation

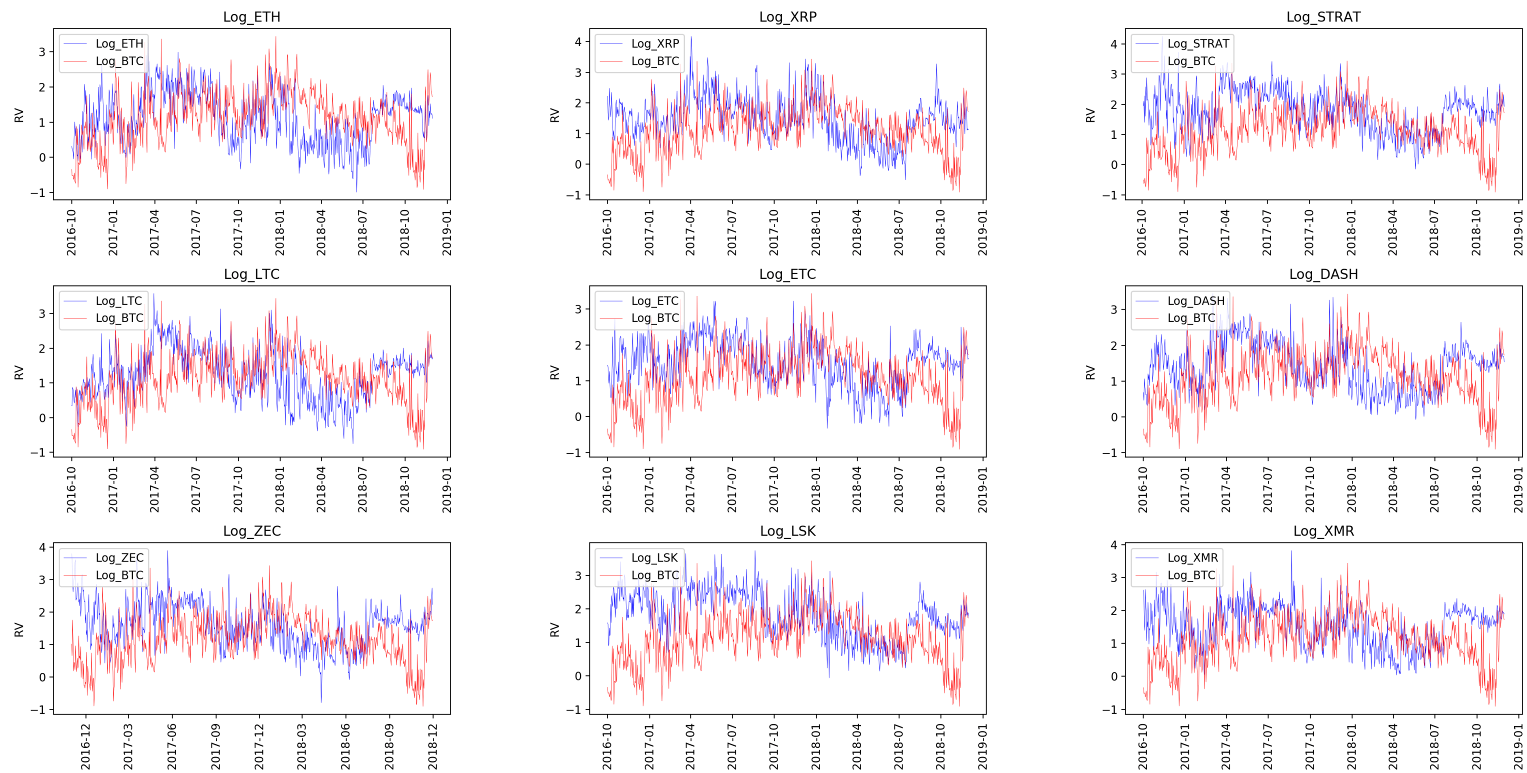

Figure 4: Log RV_t

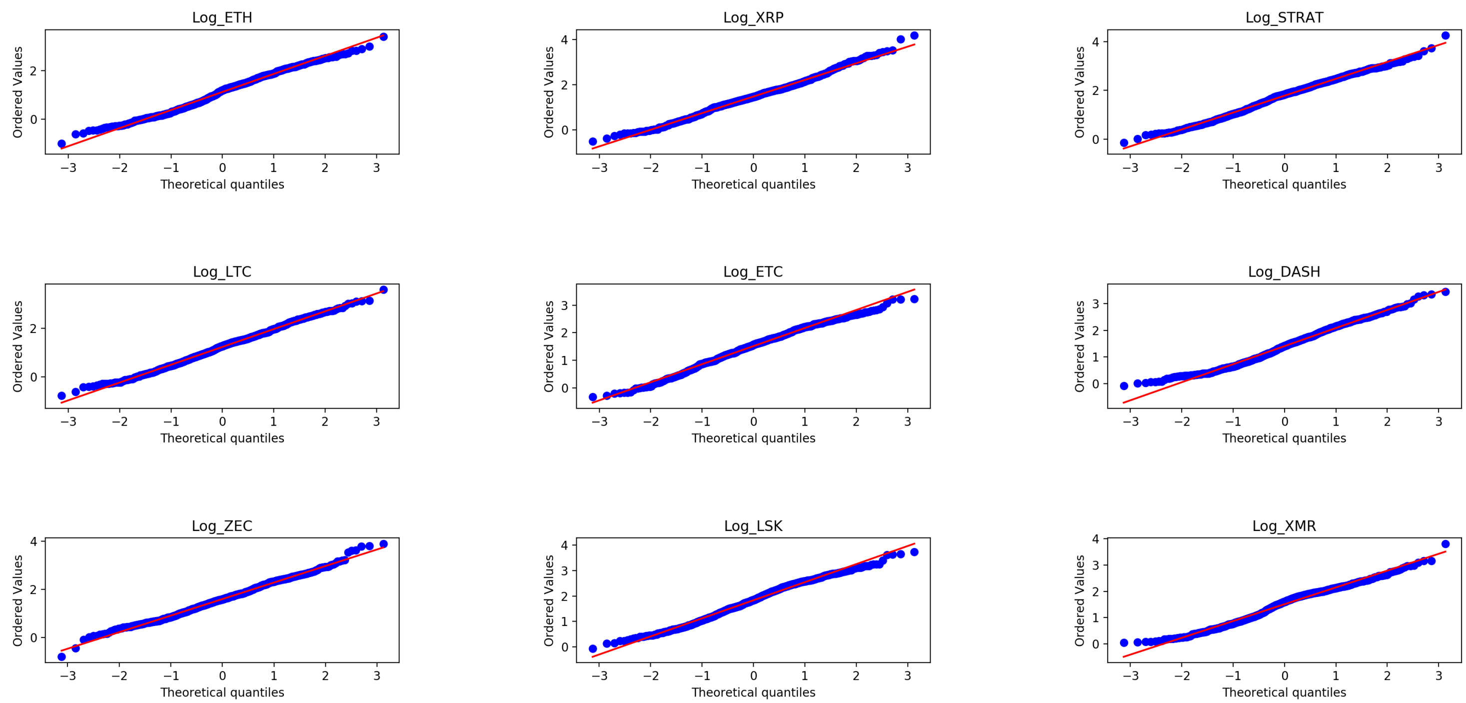

Figure 5: Log RV_t QQ Plots

Table 3: Sample Statistics of Cryptocurrency Realized Volatility

Panel A : Sample Statistics of Cryptocurrency RV_t

This panel shows sample statistics for daily realized volatility for all cryptocurrency during the October 2016-November 2018 period. ACF(1) denotes the first-order autocorrelation. Q(L) is the Ljung-Box test of zero autocorrelation in lags 1 through L. An asterisk indicated at the 1% level.

Statistics	ETH	XRP	STRAT	LTC	ETC	DASH	ZEC	LSK	XMR	BTC
Mean	4.02	5.82	7.48	4.46	5.57	5.16	6.31	7.96	5.52	4.02
Std.	3.12	5.51	5.62	3.60	3.57	3.89	5.17	5.65	3.72	3.33
Min	0.37	0.60	0.86	0.47	0.72	0.93	0.45	0.94	1.04	0.40
25%	1.71	2.83	3.59	2.04	2.95	2.37	3.01	3.73	2.68	1.96
50%	3.30	4.33	6.26	3.55	4.80	4.11	4.92	6.36	4.98	3.17
75%	5.40	6.74	9.76	5.55	7.25	6.74	8.00	11.08	7.24	5.08
Max	29.80	65.14	70.14	35.79	25.18	31.54	49.42	42.19	45.48	31.09
Skewness	2.02	4.04	3.08	2.45	1.47	2.13	3.26	1.62	2.68	2.93
Kurtosis	10.72	31.04	25.36	13.62	6.60	10.33	21.04	7.44	21.68	17.43
ACF(1)	0.72*	0.68*	0.62*	0.72*	0.72*	0.72*	0.68*	0.68*	0.58*	0.57*
Q(5)	1427.45*	948.94*	1096.56*	1564.51*	1264.85*	1338.11*	965.29*	1211.05*	868.75*	671.98*
Q(21)	3841.71*	1875.95*	2696.37*	4257.24*	3330.80*	4547.58*	2208.61*	3755.61*	2190.55*	1491.79*

Panel B: Sample Statistics of Cryptocurrency Log RV_t

This panel shows sample statistics for log daily realized volatility for all cryptocurrency during the October 2016-November 2018 period. ACF(1) denotes the first-order autocorrelation. Q(L) is the Ljung-Box test of zero autocorrelation in lags 1 through L. An asterisk indicated at the 1% level.

Statistics	ETH	XRP	STRAT	LTC	ETC	DASH	ZEC	LSK	XMR	BTC
Mean	1.12	1.48	1.78	1.23	1.52	1.40	1.60	1.83	1.51	1.12
Std.	0.74	0.73	0.69	0.73	0.66	0.68	0.69	0.71	0.65	0.75
Min	-0.99	-0.51	-0.15	-0.75	-0.33	-0.08	-0.80	-0.06	0.04	-0.91
25%	0.54	1.04	1.28	0.71	1.08	0.86	1.10	1.32	0.99	0.67
50%	1.19	1.47	1.83	1.27	1.57	1.41	1.59	1.85	1.61	1.15
75%	1.69	1.91	2.28	1.71	1.98	1.91	2.08	2.41	1.98	1.63
Max	3.39	4.18	4.25	3.58	3.23	3.45	3.90	3.74	3.82	3.44
Skewness	0.00	0.16	-0.10	0.04	-0.28	0.18	0.11	-0.13	-0.13	-0.17
Kurtosis	2.31	3.24	2.67	2.65	2.72	2.36	2.97	2.33	2.45	3.10
ACF(1)	0.81*	0.80*	0.78*	0.81*	0.77*	0.82*	0.77*	0.80*	0.76*	0.75*
Q(5)	1893.77*	1751.34*	1789.71*	1965.42*	1666.36*	2037.87*	1571.92*	1956.61*	1636.39*	1570.17*
Q(21)	5552.37*	5050.21*	5386.38*	6020.58*	4680.87*	6897.97*	4237.17*	6592.26*	4734.48*	3826.03*

Fact 2: *The distribution of the logarithm of realized volatility of cryptocurrencies is close to normal.*

3 Factor Structure in Cryptocurrency Returns and Volatility

We investigate the multivariate properties of cryptocurrency returns and volatilities by constructing a factor structure model in the cross-section of cryptocurrencies. Cross sectional common factors in cryptocurrencies in either returns or volatility have not been addressed in the literature. Following Liu and Tsyvinski (2018), who find that cryptocurrency returns are not exposed to stock market or macroeconomic factors, we test whether cross-currency structures in the cryptocurrency market can be explained by factors derived from the cryptocurrency rather than these exogenous factors.

3.1 A Common Factor in Cryptocurrency Returns?

To get a first impression of cross-sectional cryptocurrency dependence, Table 4 presents the correlation matrix of daily returns across cryptocurrencies in our sample period. The pairwise correlation between two cryptocurrencies' daily return ranges from 15% to 52%. It should be noted that the XRP and STRAT have relatively low average correlations, 22% and 28% respectively. The other cryptocurrencies have a similar average correlation of around 35%. The average return across all pairs of cryptocurrencies is 32%. The correlation of daily returns between each of the nine cryptocurrencies against Bitcoin and the BTC-USD return is always negative and relatively small, ranging from -2% to -19%. The negative correlation between Bitcoin and the other nine cryptocurrencies is not surprising due to Bitcoin being the counter currency of each of the cryptos. Therefore, the higher the value of Bitcoin, the higher the Bitcoin return and, since a base cryptocurrency uses Bitcoin as the counter currency, the lower the return of the crypto.

We next conduct principal component analysis to look for evidence of a common factor in our nine cryptocurrency returns. Figure 6 plots the first four principal components (PCs). These components explain 39.81%, 10.64%, 10.05%, 8.67% respectively. Figure 7 is the plot of cumulative explained ratio by the first four PCs for a total 69.17% of the cross-sectional variation in the nine cryptocurrency returns. Recent studies find evidence of a factor structure in the returns of a cross-section of commodities. For instance, Szymanowska et al. (2014) and Bakshi et al. (2019) work on the portfolio level of commodity futures and find a factor structure, arguing that the major principal components can explain the variation of commodity portfolio return and risk premia from different sorting strategies.

Table 4: Correlation Matrix of Cryptocurrency Return

The table shows Pearson correlations for all cryptocurrency daily log returns during October 2016 – November 2018 sample period. We also report the average pair correlation across each cryptocurrency and the average correlation across all pairwise correlation between two cryptocurrencies.

	ETH	XRP	STRAT	LTC	ETC	DASH	ZEC	LSK	XMR	BTC
ETH		0.26	0.31	0.33	0.52	0.38	0.40	0.37	0.41	-0.19
XRP			0.18	0.30	0.20	0.15	0.23	0.17	0.26	-0.16
STRAT				0.24	0.33	0.27	0.24	0.33	0.36	0.04
LTC					0.40	0.28	0.24	0.28	0.29	-0.10
ETC						0.31	0.32	0.47	0.30	-0.12
DASH							0.44	0.35	0.47	-0.17
ZEC								0.31	0.39	-0.15
LSK									0.31	-0.02
XMR										-0.10
Average	0.37	0.22	0.28	0.30	0.36	0.33	0.32	0.32	0.35	-0.11
All Pair Average	0.32									

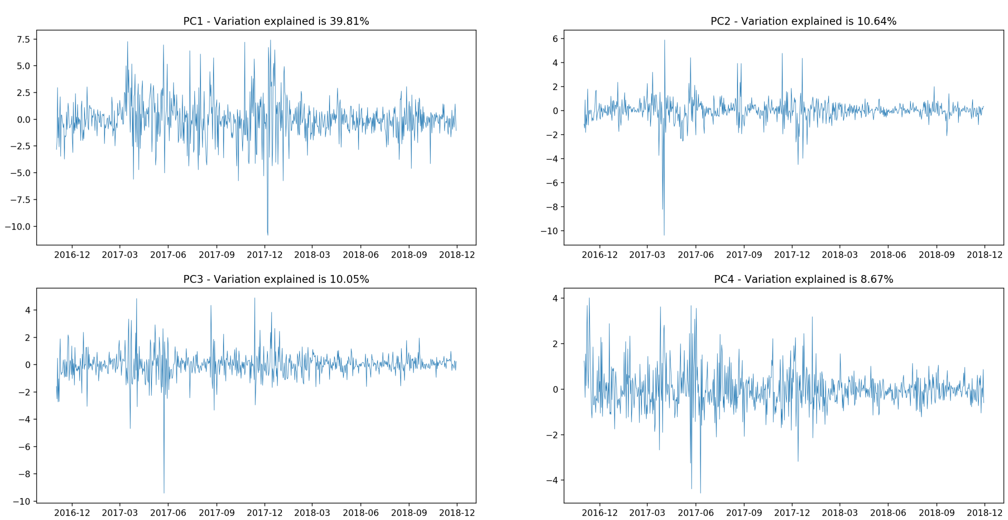

Figure 6: First Four Principle Components of Cryptocurrency Return

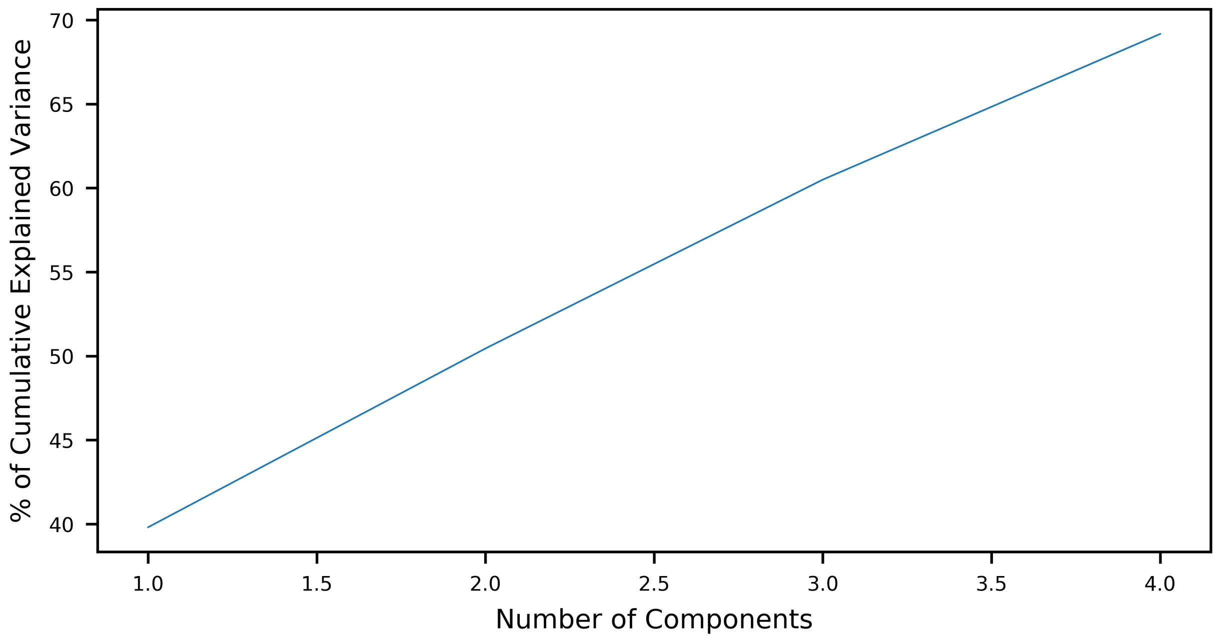

Figure 7: Cumulative Explained Variance for the First Four Principle Components of Cryptocurrency Return

Christoffersen et al. (2019) also look at commodity futures and find relatively weak evidence of a factor structure in daily commodity future returns. In their study, the first four PCs can explain 65.3% variation of the cross-section of 15 commodities' daily return, which is close to our cryptocurrency finding of about 70%. Nevertheless, the first principal component from the cryptocurrencies' returns is almost 40%, which is 10% higher than in their commodity universe. We interpret this as evidence of a factor structure in daily cryptocurrency returns and propose that a factor structure in cross-sectional cryptocurrency return has a considerable amount of pricing explanatory power in this market.

3.2 A Common Factor in Cryptocurrency Volatility?

Evidence that the factor structure of volatility is stronger than the factor structure of returns has been addressed in finance studies.[9] Therefore, we now question whether a factor structure of volatility exists in cryptocurrencies and, if so, whether it is more powerful than the factor structure

[9]Factor structure of idiosyncratic volatility in the equity market had been addressed by Chen and Petkova (2012), Duarte et al. (2014) and Herskovic et al. (2016); Factor structure of volatility in the commodity market is touched upon by Christoffersen et al. (2019).

Table 5: Correlation Matrix of Cryptocurrency Log RV_t

The table shows Pearson correlations for all cryptocurrency daily log realized volatility during October 2016 – November 2018 sample period. We also report the average pair correlation across each cryptocurrency and the average correlation across all pairwise correlation between two cryptocurrencies.

	ETH	XRP	STRAT	LTC	ETC	DASH	ZEC	LSK	XMR	BTC
ETH		0.71	0.64	0.75	0.74	0.74	0.63	0.67	0.70	0.16
XRP			0.63	0.70	0.67	0.63	0.55	0.66	0.60	0.23
STRAT				0.65	0.65	0.65	0.61	0.67	0.62	0.07
LTC					0.75	0.74	0.57	0.64	0.65	0.12
ETC						0.70	0.65	0.66	0.64	0.12
DASH							0.67	0.69	0.72	0.07
ZEC								0.63	0.67	0.06
LSK									0.64	0.10
XMR										0.06
Average	0.70	0.64	0.64	0.68	0.68	0.69	0.62	0.66	0.65	0.11
All Pair Average	0.66									

in cryptocurrency returns.

We investigate the multivariate properties of nine cryptocurrencies' $log(RV_t)$. Table 5 gives the correlations for log volatility of cryptocurrency. There is clear evidence that volatility has much higher correlations compared to returns. In particular, XRP and STRAT have the lowest average correlations of returns, but have an appreciable correlation of $log(RV_t)$, averaging 0.64 for both of them. The average correlations across different cryptocurrencies range from 62% to 70%. The average all pair correlation of $log(RV_t)$ is 66% compared with just 32% for returns. In addition, we check the correlation between the nine cryptocurrencies and Bitcoin. The correlation ranges from 6% to 23%, averaging 11%. In summary, there is weak correlation of log realized volatility between cryptocurrencies and their counter currency Bitcoin. Nevertheless, the weak positive correlations lead us to question the explanatory power of Bitcoin on common factors of cryptocurrency realized volatility and returns.

Figure 8 shows that the first four principal components of nine cryptocurrencies' $log(RV_t)$ capture 70.15%, 5.93%, 4.85%, and 3.87% respectively, for a total of 84.8% of the total variation as shown in Figure 9. A closer look reveals that the first principal component of $log(RV_t)$ in Figure 8 mirrors closely the time series of ETH $log(RV_t)$ in the top left panel of Figure 4.

Further, to investigate the factor structure of cryptocurrency returns and volatility, we conduct regression analyses of returns and volatility on their respective PCs. Panel A in Table 6 is a regression of each cryptocurrency return on the first four PCs. For each cryptocurrency, we re-conduct a principal component analysis based only on the other eight cryptocurrencies, to avoid mechanical correlations in the regressions. The first PC captures the most variation of cryptocurrency returns, and the other PCs are either marginally significant or insignificant in explaining the commonality

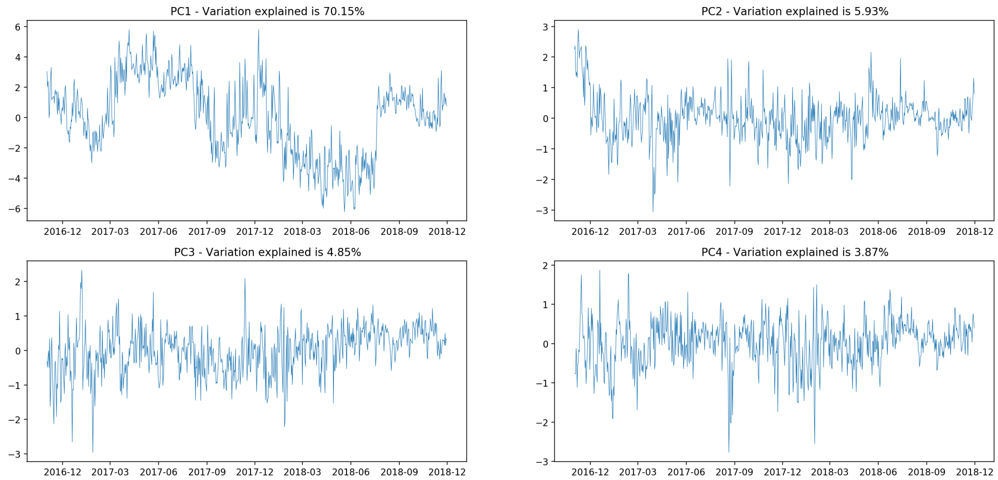

Figure 8: First Four Principle Components of Log RV_t

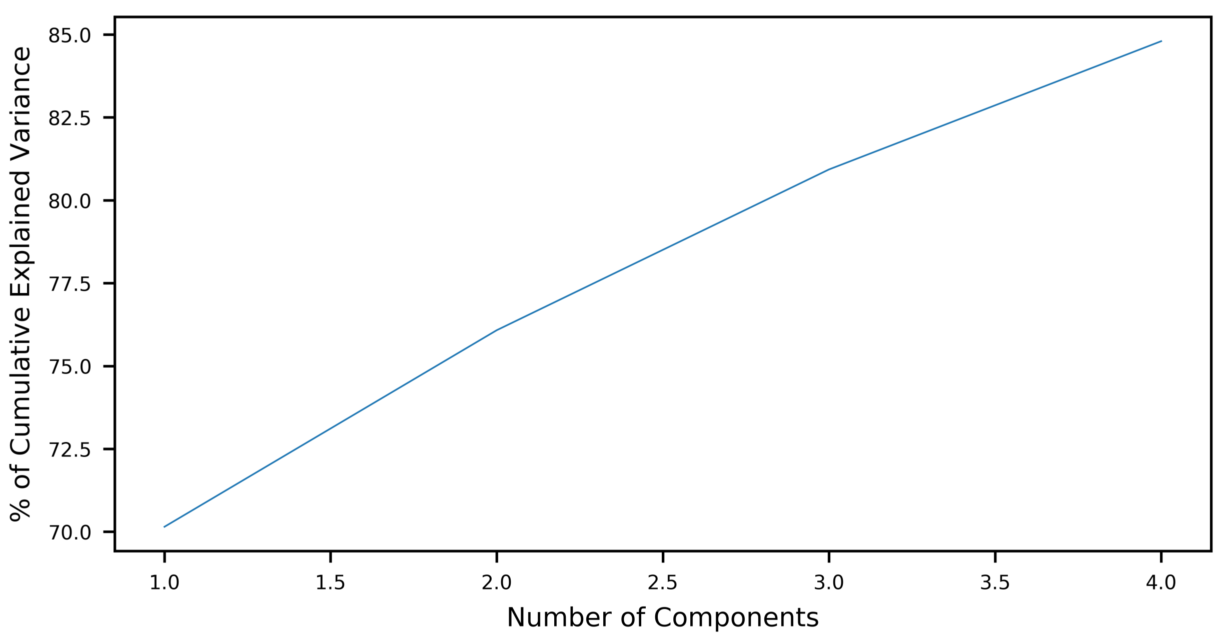

Figure 9: Cumulative Explained Variance for the First Four Principle Components of Log RV_t

Table 6: Factor Structure of the First Four Principle Components

Panel A: Regression of Daily Log Return on Principal Components

The panel shows parameter estimates of daily return regressed on principal components of 9 cryptocurrencies during the October 2016 - November 2018. For each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions.

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	1.790	13.219	-0.059	-0.241	-0.231	-1.034	-0.364	-1.325	0.366
XRP	1.578	6.355	-0.295	-0.492	-0.488	-1.192	-1.299	-2.032	0.127
STRAT	1.900	8.045	0.109	0.317	-0.271	-0.654	-0.025	-0.056	0.201
LTC	1.396	7.831	-0.264	-1.047	-0.765	-2.395	-0.225	-0.734	0.230
ETC	2.011	16.986	-0.146	-0.523	0.905	2.725	-1.374	-2.329	0.377
DASH	1.695	11.668	0.337	0.718	1.064	2.999	-0.603	-1.755	0.323
ZEC	2.048	18.334	0.346	1.411	0.892	2.736	-0.841	-2.908	0.289
LSK	2.092	10.378	0.146	0.345	-1.081	-2.271	-0.034	-0.094	0.287
XMR	1.699	15.134	0.236	0.676	-0.767	-2.343	0.766	1.891	0.335

Panel B: Regression of Daily Log RV_t on Principal Components

The panel shows parameter estimates of daily log realized volatility regressed on principal components of 9 cryptocurrencies during the October 2016 - November 2018. For each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions.

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.263	29.441	-0.073	-1.834	0.113	3.112	-0.013	-0.303	0.712
XRP	0.237	20.306	0.129	3.628	-0.071	-1.786	0.005	0.111	0.601
STRAT	0.221	22.391	-0.011	-0.253	0.086	2.031	-0.007	-0.191	0.578
LTC	0.251	24.905	-0.138	-3.187	0.118	3.862	-0.009	-0.243	0.693
ETC	0.227	23.229	-0.052	-2.188	0.038	1.179	0.008	0.239	0.673
DASH	0.241	30.340	-0.041	-1.444	0.075	2.885	0.008	0.209	0.698
ZEC	0.210	16.858	-0.153	-3.248	0.068	2.046	-0.001	-0.026	0.567
LSK	0.235	24.460	0.023	0.752	-0.086	-2.312	-0.002	-0.040	0.618
XMR	0.210	25.509	-0.080	-2.169	0.075	2.202	-0.005	-0.111	0.615

of returns. The average of R^2 is about 28%.

Panel B in Table 6 shows the regression of each cryptocurrency volatility on the first four PCs, which are again recomputed using the other eight cryptos. While the first PC captures the most variation of cryptos' volatility, the second and third PCs also capture appreciable amounts of volatility variation. The fourth PC is insignificant. All R^2 calculations from each crypto volatility regression are much higher than the R^2 in the return analysis. Noticeably, the average R^2 in volatility analysis is about 64% compared with just 28% in returns. In sum, commonality in volatility is much greater than commonality in returns. We conclude that:

Fact 3: *The factor structure in daily cryptocurrency volatility is stronger than the factor structure in returns.*

4 Economic Factors and Cryptocurrency Commonality

In this section, we investigate whether the common factors of cryptocurrency return and volatility are related to economic and financial factors. In particular, we study return and volatility from S&P500 E-mini futures, Gold futures, Crude Oil futures, CBOE SPX VIX and the spot rate of foreign exchange currencies including CNH/USD and EUR/USD. The calculation of returns and volatilities on economic and financial factors is discussed in section 2 above.

4.1 Impact of Economic Factors on Cryptocurrency Return and Volatility

In section 3, we studied the factor structure of cryptocurrency returns and realized volatility. While much more pronounced for volatility, there is still a clear factor structure in crypto returns. We now investigate whether or not the time-series of the key principal components of cryptocurrency return and volatility can be explained by fundamental economic and financial factors. For this study, we regress each PC on each economic factor as follows:

$$PC_{i,t} = \alpha + \beta_1 X_t + \beta_2 PC_{i,t-1} + \varepsilon_{i,t} \tag{6}$$

Considering potential spurious regression problems, we also add the lagged PC as an additional controlled regressor in the model. We seek to determine whether β_1, the estimated regression coefficient on the economic factor, provides a significant explanatory power for the variation in the PCs. Since cryptocurrency has a 24-7 continuous trading pattern and products of economic factors are not traded over the weekend and on Federal holidays, we merge data which are subject to economic factor trading rules.[10] Except for the two foreign exchange currencies, all economic factor data are available from October 2016 to November 2018 and all factors are available with daily frequency.

Table 7 reports regression results from equation (6). Panel A in Table 7 reports results for the PCs of cryptocurrency returns on economic factors. The first principal component is marginally significantly related to returns on the S&P500 E-mini future, but the R^2 value is quite low. The other economic factors have no significant relationship to the time series variation of cryptocurrency return PCs, and all regression R^2 values are low. This lack of a relationship between the key components of crypto returns and economic or financial factors is consistent with extant studies (Yermack, 2015; Liu and Tsyvinski, 2018; Biais et al., 2019).

Panel B in Table 7 repeats the analysis using PCs of cryptocurrency realized volatility. The first key finding is the significant negative relationship between the first PC of crypto volatility and both the volatility of the S&P500 E-mini future and the CBOE SPX VIX. In addition, the first principal component is also marginally negatively related to the volatility of CNY/USD and EUR/USD exchange rates.

The reason for the negative relationship between cryptocurrency volatility factors and macroeconomic indices (S&P500, VIX) is not clear. One potential explanation is that cryptocurrency is more susceptible to investor sentiment than macroeconomic factors, although the latter may influence the former (see for example Chuen et al. (2017), Corbet et al. (2018), and Drobetz et al. (2019)). High macroeconomic risk leads to more caution amongst investors and, as a consequence, less trading activity. As less trading activity results in less irrational trading, the volatility of cryptocurrencies in particular tends to decline. We leave the true underlying reason for a negative relationship between crypto volatility and macro volatility for a future study.

The negative relationship between commonality cryptocurrency volatility and that of foreign exchange is also not clear-cut. One possible reason is that cryptocurrency ultimately needs to be converted to fiat currency for at least some investors. If the major foreign exchange rates are highly volatile, cryptocurrency traders are reluctant to trade more. As a result, cryptocurrency becomes less volatile for the same reasons as outlined above.

In sum, there is strong evidence to show that both daily returns and realized volatilities of cryptocurrency cannot be explained by traditional economic factors. It is not surprising that there is almost no significant relation between cryptocurrency return and benchmark economic factor returns, as this has been addressed in extant studies of cryptocurrency returns. The lack of explanatory power for realized volatilities in cryptocurrencies contrasts with findings in other finan-

[10] The regression result is not sensitive to the data merging method.

Table 7: Economic Factors Impact on Principle Components of Cryptocurrency

Panel A: Regression of Principal Components of Cryptocurrency Return on Economic Factors

The table shows output from the regression of the first four principal components of cryptocurrency return on its lags and different economic return factors. S&P500 is daily log return calculated from S&P500 E-Mini Futures. Gold is daily log return calculated from Gold Futures traded. Oil is daily log return calculated from Crude Oil WTI Futures. CNHUSD is daily log return calculated from the daily spot rate of CNH/USD. EURUSD is daily log return calculated from the daily spot rate of EUR/USD. VIX is the daily log return calculated from COBE SPX Volatility Index. All factor minutely data is downloaded from TRTH with sample period from October 2016 – November 2018 except CNHUSD and EURUSD which are not available before February 2017. The principal components are constructed as the matrix of the log return for all 9 cryptocurrencies multiplied by the eigenvectors of the covariance matrix.

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**S&P500**	**S&P500-t**	R^2
PC1	-0.003	-0.035	0.104	2.014	0.142	1.945	0.014
PC2	0.004	0.112	-0.017	-0.274	-0.013	-0.289	0.000
PC3	0.001	0.036	0.114	1.538	-0.021	-0.516	0.013
PC4	-0.004	-0.093	0.003	0.036	0.017	0.364	0.000

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**Gold**	**Gold-t**	R^2
PC1	0.006	0.076	0.105	2.060	0.034	0.285	0.011
PC2	0.003	0.087	-0.016	-0.257	-0.009	-0.153	0.000
PC3	0.001	0.015	0.113	1.559	0.008	0.145	0.013
PC4	-0.004	-0.105	0.006	0.065	-0.051	-0.589	0.002

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**Oil**	**Oil-t**	R^2
PC1	0.004	0.053	0.107	2.056	0.041	0.854	0.012
PC2	0.003	0.077	-0.018	-0.284	0.034	1.595	0.004
PC3	0.001	0.014	0.113	1.556	-0.011	-0.581	0.013
PC4	-0.003	-0.068	0.007	0.073	-0.001	-0.062	0.000

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**CNHUSD**	**CNHUSD-t**	R^2
PC1	0.004	0.053	0.107	2.056	0.041	0.854	0.012
PC2	0.000	0.011	0.046	0.724	-0.196	-1.640	0.005
PC3	-0.003	-0.066	0.053	0.953	-0.030	-0.244	0.003
PC4	-0.001	-0.023	-0.088	-1.314	-0.077	-0.719	0.009

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**EURUSD**	**EURUSD-t**	R^2
PC1	-0.004	-0.043	0.090	1.717	0.100	0.556	0.009
PC2	0.000	-0.001	0.041	0.619	0.010	0.111	0.002
PC3	-0.004	-0.099	0.053	0.950	0.104	1.041	0.005
PC4	-0.001	-0.025	-0.089	-1.325	-0.005	-0.062	0.008

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**VIX**	**VIX-t**	R^2
PC1	0.005	0.067	0.100	1.885	0.001	0.079	0.010
PC2	0.005	0.120	-0.027	-0.473	0.003	0.700	0.001
PC3	0.001	0.028	0.093	1.394	0.000	-0.034	0.009
PC4	-0.004	-0.104	-0.019	-0.223	-0.004	-0.832	0.002

Panel B: Regression of Principal Components of Cryptocurrency Log RV_t on Economic Factors

The table shows output from the regression of the first four principal components of cryptocurrency log realized volatility on its lags and different economic volatility factors. S&P500 is daily log realized volatility calculated from S&P500 E-Mini Futures. Gold is the daily log realized volatility calculated from Gold Futures. Oil is daily log realized volatility calculated from Crude Oil WTI Futures. CNHUSD is daily log realized volatility calculated from daily spot rate of CNH/USD. EURUSD is daily log realized volatility calculated from daily spot rate of EUR/USD. VIX is the daily log realized volatility calculated from COBE SPX Volatility Index. All factor minutely data is downloaded from TRTH with sample period from October 2016 – November 2018 except CNHUSD and EURUSD which are not available before February 2017. The realized volatility calculation is subject to Hansen and Lunde (2005) method described under section 2. The principal components are constructed as the matrix of the log realized volatility for all 9 cryptocurrencies multiplied by the eigenvectors of the covariance matrix.

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**S&P500**	**S&P500-t**	R^2
PC1	-0.182	-2.703	0.868	45.488	-0.281	-3.330	0.789
PC2	0.028	0.677	0.610	10.189	0.051	1.000	0.379
PC3	0.014	0.289	0.456	8.433	0.023	0.349	0.211
PC4	0.018	0.474	0.448	9.343	0.028	0.689	0.201

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**Gold**	**Gold-t**	R^2
PC1	0.009	0.111	0.883	47.442	0.028	0.178	0.781
PC2	0.080	1.139	0.597	11.097	0.194	1.473	0.382
PC3	-0.048	-0.800	0.453	8.530	-0.113	-1.011	0.213
PC4	-0.064	-1.290	0.446	9.541	-0.150	-1.672	0.209

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**Oil**	**Oil-t**	R^2
PC1	-0.017	-0.232	0.883	47.089	0.033	0.217	0.781
PC2	-0.081	-1.925	0.596	10.695	0.190	2.071	0.382
PC3	0.028	0.629	0.460	8.731	-0.067	-0.741	0.212
PC4	-0.004	-0.096	0.452	9.883	0.012	0.149	0.204

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**CNYUSD**	**CNYUSD-t**	R^2
PC1	-0.283	-1.669	0.868	39.438	-0.208	-1.682	0.781
PC2	-0.121	-1.372	0.500	9.903	-0.088	-1.385	0.257
PC3	-0.150	-1.819	0.396	6.925	-0.109	-1.961	0.169
PC4	0.021	0.255	0.472	7.768	0.014	0.243	0.224

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**EURUSD**	**EURUSD-t**	R^2
PC1	-0.265	-1.659	0.878	42.832	-0.323	-1.791	0.781
PC2	-0.132	-1.437	0.502	9.909	-0.158	-1.384	0.257
PC3	-0.183	-1.893	0.394	6.899	-0.220	-2.001	0.172
PC4	-0.097	-1.108	0.471	7.743	-0.118	-1.195	0.226

	Constant	**Constant-t**	PC_{t-1}	$PC-t_{t-1}$	**VIX**	**VIX-t**	R^2
PC1	0.402	2.164	0.876	45.798	-0.210	-2.247	0.780
PC2	-0.156	-1.113	0.611	10.245	0.079	1.153	0.379
PC3	-0.078	-0.614	0.454	8.625	0.041	0.659	0.209
PC4	-0.011	-0.128	0.471	11.332	0.006	0.157	0.223

cial markets. Christoffersen et al. (2019) state that most of the macro factors they consider have a strong relation to the first component of cross-section commodity futures realized volatility. Based on their empirical evidence, the R^2 in the regression of the first PC of $log(RV_t)$ is around 70%. Comparable regressions in cryptocurrency show that the PCs can only be explained by their first lags, and not by the economic factors. The relatively high R^2 in the realized volatility PCs regression derives mainly from the lagged variable and the factor structure itself, with very little contributed by the economic or financial factors.

Therefore, despite the presence of some significant correlations, the overall relationship between macro factors and the PCs of cryptocurrency return and volatility remains relatively weak and we conclude that:

Fact 4: *Economic and financial factors do not have strong explanatory power on the common factors of cryptocurrency return and volatility and there is a weak inverse relation between risk of cryptocurrency and macroeconomic indices.*

5 Bitcoin Impact on Cryptocurrency Return and Volatility

5.1 Bitcoin as a Fundamental Factor in the Cryptocurrency market

In this section, we investigate whether the behavior of Bitcoin can be thought of as a fundamental factor to explain the time series variation of the PCs of cryptocurrency return and volatility. Products traded on Bittrex are mainly cryptocurrencies quoted against Bitcoin. This trading feature is very similar to a foreign exchange, as one is the base currency and the other is the counter currency. Bitcoin as a counter currency is a very liquid product in the crypto markets and it is reasonable to hypothesize that fluctuations in the Bitcoin price against the dollar have an impact on other cryptocurrencies.

On average, Bitcoin return has a weak negative correlation with all other cryptocurrency returns shown in Table 4. In the meantime, the $log(RV_t)$ of Bitcoin is positively correlated with other cryptocurrency $log(RV_t)$ shown in Table 5. We first run regressions of the time series of Bitcoin returns and realized volatility on time series of PCs (from returns and volatility). Table 8 shows the results. The sign of regression coefficient on both first principal components is as expected and significant at the 5% level. Other PCs are also statistically significant; however, the R^2 values are quite small at just 7.6% for the return regression and 6.2% for the volatility regression, which does not suggest a strong relationship. It appears the common components of returns (or realized volatilities) in the cryptocurrencies are weakly related to Bitcoin returns (and volatility).

As an alternative approach, we test whether Bitcoin adds explanatory power over and above the principal components for the returns and volatilities of individual cryptocurrencies. For each cryptocurrency, we regress its return (or volatility) on the first four PCs and the return (or volatility) of Bitcoin. As usual, the first four PCs are computed by taking the other eight cryptocurrency returns or volatilities. We orthogonalize each PC by regressing it on the relevant Bitcoin variable and taking the residuals. This gives Bitcoin the maximum possible chance of explaining the returns and volatilities of the individual cryptocurrencies. The regression model is as follows:

$$log(Return/RV_{i,t}) = \alpha + \beta_1 PC_{i,t} + \beta_2 log(Return/RV_{BTC,t}) + \varepsilon_t \tag{7}$$

Table 9 gives the regression results. Panel A of Table 9 shows all cryptocurrency returns are significantly negatively related to the return of Bitcoin with the exceptions of STRAT and LSK. Panel B of Table 9 reports the regression results of each cryptocurrency volatility. All cryptocurrencies are significantly positively related to $log(RV_t)$ of Bitcoin, though the level of significance differs across the cryptocurrencies. Each of ETH, XRP, LTC, ETC, DASH and LSK are significant at the 1% level, XMR is significant at 5%, while STRAT and ZEC are only significant at the 10% level. Nevertheless, the goodness of fit statistics for each regression are only slightly increased from those reported in Table 6. It appears that while Bitcoin captures some information relevant to explaining returns and volatilities of cryptocurrency i, the other eight cryptocurrencies themselves already contain much of this information already.

So far, we conclude that:

Fact 5: *Bitcoin can be considered for most cryptocurrencies as a fundamental factor able to explain a small proportion of the variations in return and volatility.*

5.2 Bitcoin Bubble Impact in Cryptocurrency Return and Volatility

In our sample period, the Bitcoin price from the Coinbase exchange climbed from $615.65 on October 1, 2016 to the peak price of $19650.01 on December 16, 2017 (see Figure 10). The Bitcoin price increased almost 32-fold in only 6 months. After reaching its peak price, Bitcoin tumbled until February 2018. On February 5, 2018 the price closed at $6905.19, its lowest point after the price peak. It is important to examine the impact of the Bitcoin bubble on the behavior of other cryptocurrencies. The weak relationship highlighted above between Bitcoin and other cryptocurrencies could be due to shifts in the underlying relationships across the different Bitcoin periods.

Table 8: Regression of BTC Return and RV_t on Principal Components

Panel A in the table shows the regression of Bitcoin return and volatility on principal components of the other 9 cryptocurrencies. Panel B is the same regression with Bitcoin bubble detection. The principle components are constructed as the matrix of the log realized volatility or returns for all 9 cryptocurrencies multiplied by the eigenvectors of the covariance matrix.

Panel A : Pooled Regression of BTC return and RV on PCs from the other 9 Cryptocurrency

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
BTC Return	-0.4089	-2.554	0.2288	0.975	-0.6063	-2.901	0.8494	3.659	7.60%
BTC RV_t	0.0375	2.297	-0.1698	-3.26	-0.007	-0.095	-0.1489	-2.567	6.20%

Panel B Sub-group Regression of BTC return and RV on PCs from the other 9 Cryptocurrency

Pre-Bubble									
BTC Return	-0.719	-3.59	-0.424	-1.221	0.0406	0.0105	0.4883	2.001	16.80%
BTC RV_t	0.0539	3.515	-0.2608	-4.425	0.0813	1.272	-0.0989	-1.41	12.30%
Bubble									
BTC Return	-0.9221	-3.917	-0.8835	-2.347	0.7326	1.148	0.0793	0.205	15.80%
BTC RV_t	0.0703	2.792	-0.1337	-2.579	0.0286	0.481	-0.0084	-0.115	13.20%
Post Bubble									
BTC Return	0.7384	5.352	-0.8717	-3.361	0.8405	2.8	1.0039	3.524	24.80%
BTC RV_t	-0.028	-1.152	-0.2156	-3.884	-0.1365	0.126	-0.1194	-1.594	8.10%

Table 9: Regression of Cryptocurrency Return and RV_t on BTC Return and RV_t

Panel A in the table shows parameter estimates of return regressed on principal components of 9 cryptocurrencies and the Bitcoin daily return during the October 2016 - November 2018. Panel B in the table shows parameter estimates of log realized volatility regressed on principal components of 9 cryptocurrencies and the Bitcoin daily log realized volatility during the October 2016 - November 2018. Noted that, for each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions. Besides, we take residuals from the equation (7) as principal components from the other 8 cryptocurrencies and orthogonalize it to data from Bitcoin.

Panel A: Regression of Cryptocurrency Returns on BTC and PCs

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.116	0.758	-0.229	-5.523	1.741	13.264	-0.030	-0.128	-0.316	-1.402	-0.275	-1.023	0.377
XRP	0.353	1.163	-0.315	-5.178	1.496	6.060	-0.361	-0.606	-0.285	-0.645	-1.297	-2.079	0.138
STRAT	0.092	0.346	0.076	1.261	2.012	9.032	0.073	0.205	-0.170	-0.400	0.010	0.022	0.218
LTC	0.065	0.358	-0.129	-2.539	1.386	7.769	-0.249	-0.972	-0.768	-2.407	-0.204	-0.657	0.230
ETC	0.026	0.147	-0.163	-3.425	1.987	16.486	-0.128	-0.457	0.961	2.930	-1.365	-2.297	0.378
DASH	0.124	0.780	-0.217	-3.615	1.651	11.920	0.405	0.892	1.069	3.065	-0.509	-1.557	0.330
ZEC	-0.524	-1.798	-0.246	-6.149	2.012	18.854	0.390	1.597	0.873	2.746	-0.756	-2.533	0.292
LSK	0.051	0.222	-0.034	-0.458	2.145	10.959	0.132	0.308	-1.026	-2.217	-0.128	-0.341	0.292
XMR	0.131	0.792	-0.120	-4.011	1.702	15.376	0.233	0.665	-0.772	-2.290	0.763	1.867	0.335

Panel B: Regression of Cryptocurrency Log RV_t on BTC and PCs

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.953	18.301	0.162	4.483	0.261	28.889	-0.066	-1.726	0.113	3.151	-0.006	-0.151	0.713
XRP	1.193	27.488	0.236	6.547	0.233	19.311	0.115	3.197	-0.072	-1.904	0.011	0.253	0.619
STRAT	1.703	32.504	0.063	1.694	0.222	23.052	-0.004	-0.097	0.090	2.068	-0.007	-0.187	0.580
LTC	1.109	26.095	0.126	3.866	0.251	24.184	-0.141	-3.321	0.120	3.929	-0.009	-0.263	0.693
ETC	1.399	35.105	0.107	3.807	0.227	22.804	-0.052	-2.226	0.038	1.181	0.008	0.242	0.673
DASH	1.330	30.397	0.070	2.416	0.243	29.497	-0.036	-1.269	0.075	2.881	0.004	0.107	0.699
ZEC	1.535	28.570	0.054	1.701	0.211	16.328	-0.151	-3.281	0.067	1.965	-0.003	-0.072	0.567
LSK	1.708	27.419	0.098	2.564	0.235	23.938	0.023	0.748	-0.086	-2.305	-0.002	-0.038	0.618
XMR	1.428	32.615	0.055	1.956	0.211	24.333	-0.076	-2.094	0.074	2.170	-0.010	-0.206	0.616

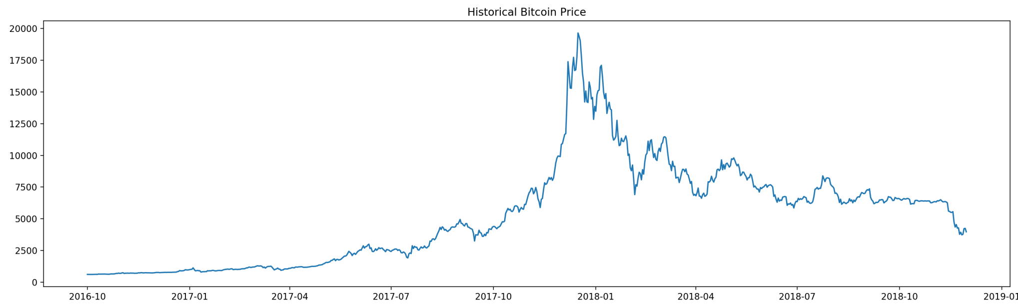

Figure 10: Historical Price of Bitcoin from Oct. 2016 to Nov. 2018

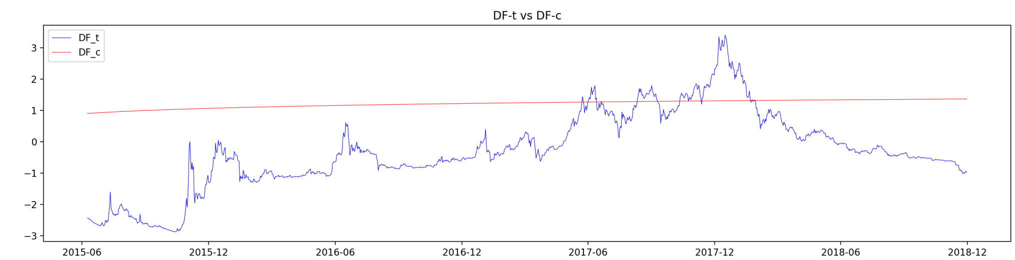

Figure 11: Bitcoin Price Bubble Test

Our initial aim is to find the bubble's origin and burst dates. We follow the approach of Phillips et al. (2011) and Phillips and Yu (2011) using forward recursive regressions to calculate Dickey-Fuller (DF) t statistics that can then be compared to the critical value of the DF test defined in their paper. Figure 11 plots the DF t statistics and critical values (see Appendix A for detailed calculations). We adopt the definition of the burst of the bubble from the Phillips et al. (2011) paper and define the bubble burst date to be the last date on which the DF statistic is greater than the DF critical value. We keep the origin of the bubble as the earliest date on which the DF statistic is greater than the DF critical value.[11] Based on this method, the Bitcoin bubble began on May 24, 2017 and ended on January 28, 2018. Our bubble period naturally includes the price peak of December 16, 2017. The bubble lasted 250 days, and contains around one-third of the data in the sample period, allowing us to define three sub-periods: the period before May 24, 2017 is defined as the pre-bubble period; May 24, 2017 through January 28, 2018 is the bubble period; and the interval after January 28, 2018 is the post-bubble period.

We now re-conduct regression analysis of Bitcoin factors in these three sub-periods. Table

[11] The test statistics often dropped below or rose above the relevant critical values between these dates. In real time, dating the bubble with this approach would have been difficult, but our interest is in historically dating the bubble solely in order to split our sample into pre-bubble, bubble and post-bubble periods.

10 reports the results of regressing cryptocurrency returns on the first four PCs.[12] There are clear indications that the factor structure is a powerful way to explain the variation in cryptocurrency returns and that bubble-related dynamics are important. All R^2 values from the bubble period are significantly higher than in the pre-bubble period. It should be noted that the XRP and STRAT R^2 values are only 5% and 8.6% respectively in the pre-bubble period and that these both increase to 25% during the bubble. Once the bubble had burst, the R^2 figures remain close to or, in some cases, above the same statistics from the bubble interval. The simple average R^2 values across nine cryptocurrencies are 18%, 40%, 36% from the pre-bubble, bubble and post-bubble respectively. In sum, the commonality in cryptocurrency returns is stronger during and, to a large extent, after the Bitcoin bubble.

Table 11 reports the regression of each cryptocurrency's volatility on the first four volatility PCs. The explanatory power of PCs in the volatility regressions using the full sample were higher than for returns, and this survives splitting the sample into sub-periods. Explanatory power again increases from the pre-bubble to the bubble period for volatility (from an average of 48 % to 56%) and continues to rise in the post-bubble period (averaging 71%). Consequently, based on the findings here, we conclude that:

Fact 6: *The Factor Structure model is more powerful in explaining variation in returns and volatilities during the Bitcoin bubble period and this explanatory power persists - and, for volatilities actually increases further - after the Bitcoin bubble burst.*

5.3 The Shifting Relationship between Variation in Cryptocurrency and Bitcoin on Returns and RV

Next, we investigate whether the Bitcoin pricing bubble affects the abilities of PCs of returns or volatility to explain the time series variation of Bitcoin returns and volatility. Panel B of Table 8 reports results from the regression of Bitcoin return (or volatility) on PCs during the three sub-periods. Compared with the full sample results given in Panel A, the sub-period results suggest considerable instability in coefficients. In particular, coefficient signs on the first PC flip in the post-bubble period for both returns and volatility. Not surprisingly, therefore, sub-period R^2 figures are much higher than the apparently mis-specified full sample regression.

The explanation for the shifting relationship between common factors of cryptocurrency and the variation in Bitcoin return and volatility is not clear. However, it does suggest that the shifting fundamental behavior of Bitcoin after the bubble burst is important. Bitcoin became considerably

[12] Again, the PCs are computed by the other eight currencies to avoid an endogeneity issue.

Table 10: Regression of Cryptocurrency Return on PCs during Pre-Bubble, Bubble, and Post Bubble Periods

The table shows parameter estimates of daily return regressed on principal components of 9 cryptocurrencies conditional on Bitcoin bubble problem during the October 2016 - November 2018. For each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions.

Panel A: Pre-Bubble

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	2.073	5.939	-0.143	-0.416	0.551	1.147	-0.955	-1.842	0.255
XRP	0.498	0.742	0.487	0.478	1.372	1.761	2.302	1.923	0.050
STRAT	1.507	3.577	0.129	0.307	-0.207	-0.493	0.794	1.328	0.086
LTC	1.095	3.291	0.669	1.052	1.082	2.421	0.405	0.764	0.103
ETC	2.229	7.872	0.103	0.204	-0.860	-2.103	-1.484	-3.936	0.348
DASH	1.529	4.268	-0.567	-0.688	0.560	1.536	1.406	2.271	0.189
ZEC	2.064	6.368	-0.310	-0.545	0.842	1.078	-1.887	-2.606	0.132
LSK	1.869	6.094	-0.078	-0.166	-0.681	-1.572	-1.466	-2.518	0.232
XMR	1.791	4.899	-0.088	-0.237	0.619	1.560	0.346	0.462	0.225

Panel B: Bubble

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	2.231	12.279	-0.523	-1.703	-0.218	-0.437	0.209	0.509	0.499
XRP	2.320	6.446	0.716	0.775	0.480	0.524	-1.181	-1.073	0.251
STRAT	2.428	5.290	-0.486	-0.540	-0.416	-0.447	-0.116	-0.128	0.248
LTC	1.841	6.386	-0.552	-1.071	0.342	0.661	-1.222	-2.362	0.337
ETC	2.548	14.651	1.105	1.592	1.958	2.500	-1.705	-1.216	0.465
DASH	2.210	10.537	0.968	1.631	0.325	0.863	-1.420	-2.202	0.422
ZEC	2.796	19.108	0.986	2.179	0.337	0.961	-0.432	-1.011	0.617
LSK	2.816	8.419	-0.600	-0.525	0.481	0.623	0.954	0.831	0.343
XMR	2.234	11.247	1.283	1.639	-1.134	-1.540	0.782	1.023	0.436

Panel C: Post-Bubble

	PC1	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	1.009	12.325	-0.346	-2.724	-0.635	-4.363	-0.249	-1.684	0.529
XRP	1.280	8.669	-0.318	-1.782	0.714	2.362	-0.263	-0.932	0.364
STRAT	1.313	13.503	-0.205	-1.064	0.401	1.545	0.448	1.819	0.379
LTC	0.838	12.442	0.171	1.162	-0.351	-2.308	0.282	1.739	0.351
ETC	0.979	9.476	0.337	1.467	-0.557	-2.053	-0.506	-1.753	0.268
DASH	1.053	9.927	0.481	2.151	-0.128	-0.881	-0.220	-1.383	0.425
ZEC	1.029	11.937	-0.317	-1.669	0.040	0.224	-0.415	-1.987	0.292
LSK	1.192	8.734	0.008	0.032	0.507	1.803	0.764	3.123	0.281
XMR	0.763	8.456	0.200	1.036	-0.603	-3.152	-0.443	-1.911	0.278

Table 11: Regression of Cryptocurrency Log RV_t on PCs during Pre-Bubble, Bubble, and Post Bubble Periods

The table shows parameter estimates of log realized volatility regressed on principal components of 9 cryptocurrencies conditional on Bitcoin bubble problem during the October 2016 - November 2018. For each cryptocurrency, we re-conduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions.

Panel A: Pre-Bubble									
	PC1	PC1-t	PC2	PC2-t	PC3	PC3-t	PC4	PC4-t	R^2
ETH	0.263	12.558	0.092	1.588	-0.131	-2.18	-0.017	-0.334	0.595
XRP	0.218	8.77	0.174	3.626	-0.078	-1.193	-0.041	-0.757	0.54
STRAT	0.172	5.784	0.075	1.158	0.035	0.383	-0.119	-1.248	0.27
LTC	0.236	11.948	-0.253	-6.622	0.157	3.386	0.097	2.369	0.647
ETC	0.17	9.468	-0.062	-2.074	-0.06	-1.285	0.031	0.756	0.457
DASH	0.281	19.436	-0.025	-0.866	-0.032	-0.673	-0.024	-0.428	0.64
ZEC	0.146	5.608	-0.242	-3.707	-0.049	-0.718	-0.141	-1.928	0.32
LSK	0.139	6.077	0.068	2.028	-0.027	-0.578	0.002	0.034	0.343
XMR	0.196	10.549	0.158	2.801	-0.036	-0.641	0.054	0.93	0.476
Panel B: Bubble									
	PC1	PC1-t	PC2	PC2-t	PC3	PC3-t	PC4	PC4-t	R^2
ETH	0.255	16.831	-0.072	-1.597	0.063	1.487	0.106	2.467	0.670
XRP	0.153	6.277	0.037	0.695	0.066	1.085	0.031	0.506	0.319
STRAT	0.142	8.856	-0.001	-0.035	-0.050	-1.059	0.000	-0.005	0.401
LTC	0.246	15.949	-0.005	-0.084	-0.029	-0.614	-0.059	-1.190	0.642
ETC	0.211	13.524	0.027	0.849	-0.046	-1.077	-0.167	-3.797	0.638
DASH	0.201	13.631	0.120	3.094	-0.148	-4.476	-0.067	-1.634	0.614
ZEC	0.205	11.441	0.071	1.942	0.007	0.228	-0.038	-1.027	0.600
LSK	0.204	15.792	-0.055	-1.558	0.034	0.619	0.144	2.293	0.571
XMR	0.194	11.643	0.017	0.284	-0.110	-2.786	-0.102	-1.503	0.569
Panel C: Post Bubble									
	PC1	PC1-t	PC2	PC2-t	PC3	PC3-t	PC4	PC4-t	R^2
ETH	0.232	22.335	0.021	0.493	-0.055	-1.285	0.007	0.134	0.759
XRP	0.213	14.561	0.103	2.570	-0.021	-0.481	-0.018	-0.270	0.660
STRAT	0.215	26.727	-0.016	-0.556	0.046	1.269	0.044	1.246	0.785
LTC	0.226	21.649	0.006	0.142	0.058	1.380	-0.101	-2.335	0.730
ETC	0.210	17.020	-0.002	-0.049	0.021	0.498	0.015	0.312	0.693
DASH	0.209	22.442	0.103	3.089	0.031	0.852	0.050	1.173	0.778
ZEC	0.182	10.803	0.088	2.141	0.039	0.933	-0.011	-0.232	0.613
LSK	0.171	12.400	0.090	2.254	-0.056	-1.122	-0.016	-0.301	0.591
XMR	0.216	24.821	0.078	1.434	0.024	0.493	0.033	0.705	0.747

Table 12: Regression of Cryptocurrency Return on PCs and BTC during Pre-Bubble, Bubble, and Post Bubble Periods

The table shows parameter estimates of daily returns regressed on principal components of 9 cryptocurrencies and the Bitcoin daily return conditional on Bitcoin bubble problem during the October 2016 - November 2018. Noted that, for each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions. Also, we take residuals from the equation (7) as principal components from the other 8 cryptocurrencies and orthogonalize it to data from Bitcoin.

Panel A: Pre-Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	1.073	2.818	-0.557	-3.495	1.893	5.705	-0.249	-0.721	0.591	1.196	-0.828	-1.818	0.268
XRP	1.787	2.386	-0.858	-4.671	-0.087	-0.128	0.446	0.513	1.540	1.963	1.956	1.597	0.093
STRAT	1.419	2.382	-0.095	-0.644	1.668	3.550	0.224	0.471	-0.236	-0.548	0.763	1.298	0.092
LTC	0.669	1.530	-0.438	-3.430	0.926	2.843	0.594	0.934	0.971	2.149	0.497	0.919	0.114
ETC	0.894	2.791	-0.397	-4.146	2.244	7.623	0.111	0.216	-0.855	-2.095	-1.494	-3.869	0.348
DASH	1.032	2.776	-0.438	-1.704	1.322	3.116	-0.679	-0.963	0.710	1.803	1.409	2.406	0.206
ZEC	-1.033	-1.060	-0.586	-2.929	1.878	6.307	-0.423	-0.776	0.906	1.163	-1.780	-2.301	0.137
LSK	0.596	1.589	-0.537	-3.565	1.638	5.536	-0.211	-0.542	-0.568	-1.379	-1.586	-3.113	0.252
XMR	0.826	2.226	-0.319	-2.285	1.769	4.304	-0.100	-0.251	0.614	1.528	0.359	0.493	0.225

Panel B: Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.373	1.364	-0.392	-6.437	2.073	10.380	-0.691	-2.334	-0.110	-0.219	0.236	0.604	0.518
XRP	0.220	0.393	-0.491	-6.135	2.144	5.460	0.583	0.610	0.340	0.341	-0.997	-0.890	0.261
STRAT	0.096	0.165	-0.166	-1.448	2.593	5.341	-0.371	-0.411	-0.325	-0.330	0.011	0.013	0.255
LTC	0.216	0.549	-0.226	-2.710	1.854	5.766	-0.539	-1.003	0.345	0.665	-1.230	-2.365	0.337
ETC	-0.032	-0.088	-0.311	-4.304	2.492	11.685	1.169	1.677	1.978	2.549	-1.720	-1.222	0.466
DASH	0.306	1.061	-0.417	-4.558	2.056	10.964	0.811	1.563	0.430	1.095	-1.466	-2.269	0.437
ZEC	-0.096	-0.349	-0.429	-9.204	2.703	15.862	0.897	1.995	0.404	1.162	-0.426	-1.011	0.622
LSK	0.703	1.388	-0.187	-1.498	3.024	9.185	-0.457	-0.416	0.351	0.455	1.005	0.878	0.355
XMR	0.267	0.772	-0.339	-7.185	2.206	10.587	1.245	1.571	-1.137	-1.545	0.780	1.026	0.437

Panel C: Post Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	-0.404	-3.002	0.119	3.013	1.026	12.530	-0.339	-2.567	-0.618	-3.810	-0.225	-1.558	0.530
XRP	-0.034	-0.141	0.149	2.485	1.320	9.671	-0.290	-1.612	0.657	2.107	-0.301	-1.097	0.366
STRAT	-0.472	-2.641	0.473	10.415	1.111	9.401	-0.001	-0.005	0.221	0.929	0.174	0.702	0.451
LTC	-0.204	-1.576	0.127	3.120	0.838	11.495	0.171	1.248	-0.350	-2.042	0.282	1.816	0.351
ETC	-0.236	-1.140	0.140	2.587	0.996	9.667	0.314	1.377	-0.574	-2.148	-0.489	-1.743	0.268
DASH	-0.318	-2.434	0.145	4.617	1.113	10.245	0.575	2.580	-0.080	-0.540	-0.161	-0.972	0.432
ZEC	-0.166	-0.836	0.181	3.678	1.034	10.657	-0.324	-1.640	0.046	0.251	-0.412	-2.060	0.292
LSK	-0.435	-1.699	0.372	5.916	1.058	8.423	0.172	0.683	0.505	1.844	0.512	2.042	0.303
XMR	-0.122	-0.823	0.272	9.241	0.646	7.125	0.025	0.138	-0.640	-3.403	-0.278	-1.331	0.316

less volatile in the third quarter 2018 yet the other nine cryptocurrencies remained highly volatile.

Further analysis is required to determine whether the Bitcoin bubble had a significant impact on the relationship between Bitcoin returns and volatility and each cryptocurrency's return and volatility. Table 12 shows the regression results of the cryptocurrency returns on Bitcoin returns and the first four orthogonalized PCs during the pre-bubble, bubble and post-bubble periods. The results are broadly similar to the regressions without adding the Bitcoin return, as shown in Table 10, and the R^2 figures are barely changed.

More interestingly, we see that after the bubble bursts, the relationship between Bitcoin returns and the returns of each cryptocurrency has significantly changed. In the pre-bubble and bubble periods, the relationship is negative and significant for most cryptocurrencies except STRAT (not significant in pre-bubble and bubble periods), LSK (not significant in the bubble period). However, all cryptocurrency returns are positive and significant at the 1% level in the post-bubble period. The relatively weak relationship noted above for the full sample regression is in part due to this structural shift.

Table 13 reports the results of volatility regression considering $log(RV_t)$ of Bitcoin and the first four orthogonalized PCs. Not all cryptocurrency $log(RV_t)$ are significantly related to Bitcoin RV before the bubble. For example, STRAT is not significant at all and LSK is only positively significant at the 10% level. The nature of the positive relationship between Bitcoin volatility and other cryptocurrencies strengthens during the Bitcoin bubble period and all cryptocurrencies' volatilities are strongly positively significant at the 1% level. Again, though, we see that the relationship between Bitcoin volatility and that of other cryptocurrencies is reversed and less significant after the bubble burst.

We also see that the change in the relationships for return and volatility are inverted. Bitcoin return becomes positively related to those of the other cryptos, while Bitcoin volatility becomes negatively related with other crypto volatilities post-bubble. The reason for the relationship shifting between the Bitcoin and Cryptocurrency return and volatility as the Bitcoin pricing bubble burst is unclear. Nevertheless, we conclude that:

Fact 7: *There is heterogeneity in the relationship between Bitcoin and other cryptocurrencies for both returns and volatility after the Bitcoin pricing bubble burst.*

6 Realized Cryptocurrency Beta and Systematic Risk Ratio

In this section, we study realized covariance between the nine cryptocurrencies and Bitcoin. As we found in section 5, Bitcoin acts as a (weak) fundamental factor in addition to PCs from the cryptocurrencies. Furthermore, Bitcoin captures almost 55% of the market value in cryptocurrency. We seek to test whether the role of Bitcoin is that of a market index proxy. Therefore, we compute "market"-style betas in the cryptocurrency market using Bitcoin as the market proxy. Given the demonstrated impact of the Bitcoin pricing bubble in our sample, we compute the dynamic, model-free, realized betas with our high-frequency returns.

Table 13: Regression of Cryptocurrency Log RV_t on PCs and BTC during Pre-Bubble, Bubble, and Post Bubble Periods

The table shows parameter estimates of log realized volatility regressed on principal components of 9 cryptocurrencies and the Bitcoin log realized volatility conditional on Bitcoin bubble problem during the October 2016 - November 2018. Noted that, for each cryptocurrency, we reconduct a principal component analysis based only on the other 8 cryptocurrencies, to avoid endogeneity issues in the regressions. In addition, we take residuals from the equation (7) as principal components from the other 8 cryptocurrencies and orthogonalize it to data from Bitcoin.

Panel A: Pre-Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	1.095	11.998	0.399	5.984	0.252	11.752	0.077	1.332	-0.117	-2.085	-0.029	-0.556	0.600
XRP	1.398	25.138	0.463	8.864	0.187	6.836	0.129	2.698	-0.118	-1.951	-0.011	-0.226	0.599
STRAT	2.086	20.478	-0.046	-0.458	0.224	7.832	0.023	0.480	0.081	0.940	-0.171	-2.036	0.358
LTC	1.212	19.086	0.426	8.255	0.221	10.107	-0.227	-5.760	0.170	4.027	0.080	1.843	0.658
ETC	1.671	25.788	0.192	4.064	0.180	9.558	-0.077	-2.530	-0.074	-1.560	0.047	1.191	0.464
DASH	1.445	19.300	0.362	6.421	0.272	14.202	-0.012	-0.484	-0.022	-0.480	-0.035	-0.615	0.643
ZEC	1.814	20.439	0.146	2.326	0.132	4.455	-0.271	-4.263	-0.041	-0.603	-0.139	-1.896	0.329
LSK	2.200	31.956	0.094	1.790	0.149	6.153	0.055	1.560	-0.039	-0.832	-0.017	-0.292	0.351
XMR	1.516	20.285	0.187	3.382	0.194	9.407	0.160	2.796	-0.035	-0.615	0.053	0.916	0.476

Panel B: Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.699	5.961	0.399	5.405	0.249	16.440	-0.060	-1.303	0.059	1.497	0.105	2.651	0.675
XRP	1.158	9.885	0.379	5.344	0.138	5.463	0.022	0.394	0.057	0.899	0.038	0.615	0.353
STRAT	1.708	19.771	0.220	4.251	0.138	8.250	-0.009	-0.235	-0.049	-1.087	-0.003	-0.055	0.404
LTC	1.015	13.025	0.244	4.633	0.251	14.193	-0.013	-0.250	-0.027	-0.559	-0.060	-1.270	0.644
ETC	1.238	16.503	0.290	6.565	0.209	11.949	0.023	0.734	-0.046	-1.047	-0.165	-3.855	0.638
DASH	1.190	12.343	0.221	4.007	0.198	13.962	0.124	3.122	-0.153	-4.692	-0.068	-1.680	0.616
ZEC	1.420	12.406	0.200	3.311	0.207	12.124	0.067	1.730	0.008	0.261	-0.038	-1.018	0.601
LSK	1.806	15.333	0.180	2.810	0.209	15.481	-0.047	-1.259	0.032	0.601	0.146	2.398	0.574
XMR	1.249	14.201	0.241	4.636	0.192	10.087	0.021	0.346	-0.111	-2.863	-0.103	-1.526	0.569

Panel C: Post Bubble

	Constant	**Constant-t**	**BTC**	**BTC-t**	**PC1**	**PC1-t**	**PC2**	**PC2-t**	**PC3**	**PC3-t**	**PC4**	**PC4-t**	R^2
ETH	0.934	17.020	-0.118	-2.628	0.231	23.267	0.029	0.748	-0.051	-1.248	0.012	0.239	0.761
XRP	1.064	17.401	-0.028	-0.664	0.215	14.643	0.095	2.476	-0.016	-0.362	-0.009	-0.129	0.662
STRAT	1.434	36.552	-0.056	-2.047	0.216	27.929	-0.008	-0.252	0.052	1.431	0.051	1.457	0.787
LTC	1.054	27.154	-0.107	-2.801	0.226	21.755	0.005	0.119	0.058	1.378	-0.100	-2.280	0.731
ETC	1.230	31.106	-0.041	-0.941	0.211	17.279	-0.013	-0.279	0.029	0.660	0.014	0.276	0.696
DASH	1.283	39.734	-0.178	-6.421	0.206	23.075	0.083	2.746	0.047	1.355	0.063	1.716	0.793
ZEC	1.336	30.178	-0.067	-1.929	0.182	10.851	0.097	2.156	0.043	1.014	-0.017	-0.347	0.614
LSK	1.271	34.565	0.030	1.065	0.174	13.746	0.079	2.052	-0.037	-0.744	-0.035	-0.756	0.605
XMR	1.444	47.315	-0.168	-5.754	0.214	26.351	0.063	1.164	0.014	0.299	0.031	0.648	0.755

6.1 Realized Covariance Construction

We calculate 1-minute log returns each day based on log mid-prices. We then compute overlapping[13] 5-minute realized covariances between cryptocurrency i and Bitcoin as:

$$RCov_t^{oc} = \frac{n}{5(n-4)} \sum_{k=1}^{n-4} \widetilde{r}_{crypto,t_k} \widetilde{r}_{BTC,t_k} \tag{8}$$

After merging data for Bitcoin and cryptocurrency i, there are again long trading breaks that we solve using the Hansen and Lunde (2005) method. The close to open return for cryptocurrency i and Bitcoin are denoted by $r^{co}_{crypto,t}$ and $r^{co}_{BTC,t}$ respectively. Due to the variety of data breaks on different trading days, we use the simulated data from perfect days to calculate the optimal weights subject to different breaking timings and duration. The final calculation is as follows:

$$RCov_t = \hat{w}_1 \sum_{i=1}^{B} r^{co}_{i,crypto,t} r^{co}_{i,BTC,t} + \hat{w}_2 \sum_{i=1}^{B+1} RCov^{oc}_{i,t} \tag{9}$$

Recalling the bubble analysis in section 5, we calculate daily model-free realized betas for each cryptocurrency. We follow studies byAndersen et al. (2005) and Patton and Verardo (2012) and the realized beta is defined as:

$$R\beta_{i,t} = \frac{RCov_{i,t}}{RV_{BTC,t}} \tag{10}$$

The realized covariance $RCov_{i,t}$ is a cross-product of the intraday cryptocurrency return and the Bitcoin return estimated by either equation (4) or (5) based on whether a day has trading breaks.

Fact 8: *Cryptocurrency betas with Bitcoin were negative before the Bitcoin bubble burst but became positive after the bubble burst.*

Figure 12 plots the daily model-free realized betas for the nine cryptocurrencies. The red line in each plot is a 99% confidence interval. Figure 12 shows clearly that realized betas are negative until February 2018. As the Bitcoin bubble bursts, almost all realized betas rise towards zero before trending upwards from April 2018. Therefore, we state that:

The realized beta measures the systematic risk of a cryptocurrency in comparison to the benchmark Bitcoin as a proxy for the cryptocurrency market factor. However, the measurement of beta does not mean we can directly suggest the extent to which the variation in cryptocurrency returns is driven by the variation of Bitcoin as a fundamental factor. We follow the Christoffersen

[13] For a more detailed method regarding overlapping trading spans, see Barndorff-Nielsen and Shephard (2004).

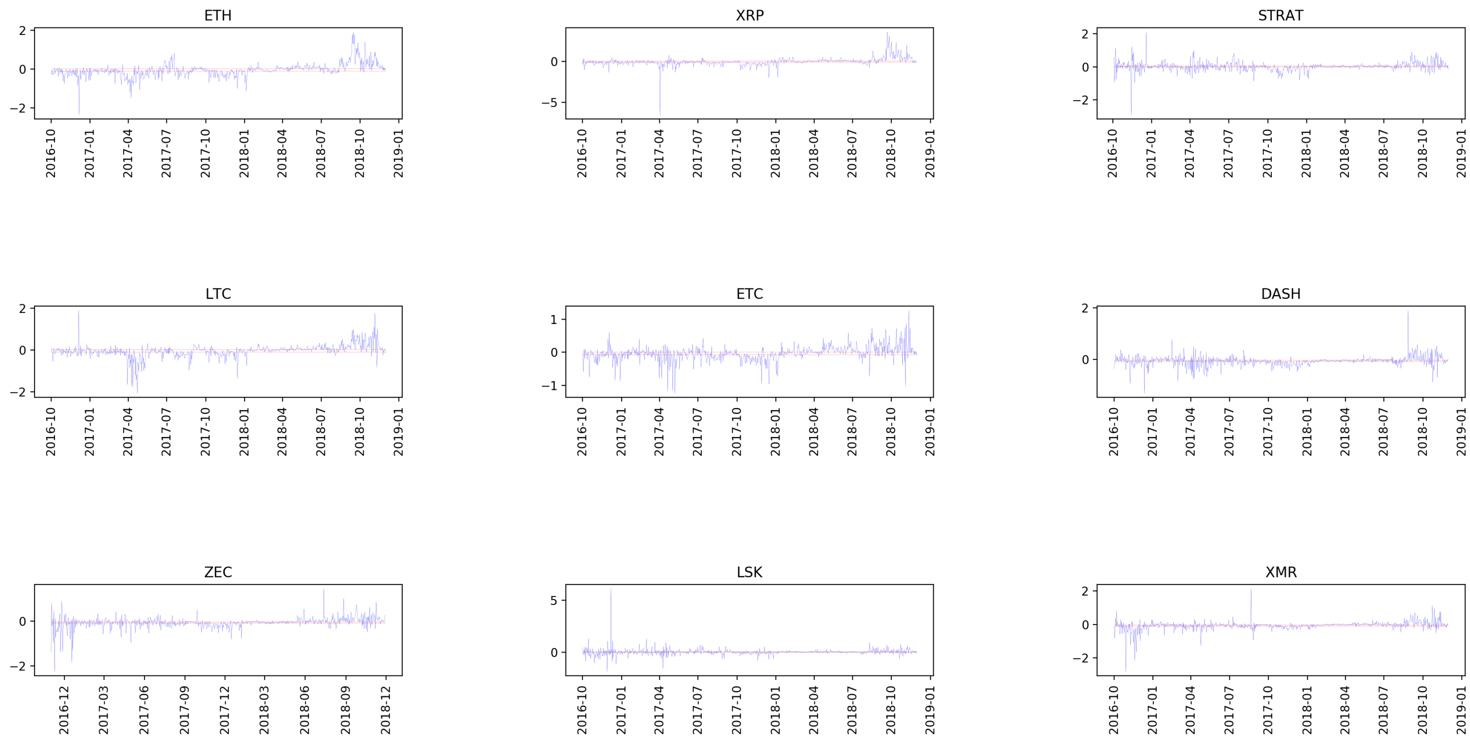

Figure 12: Cyrptocurreny Realized Betas

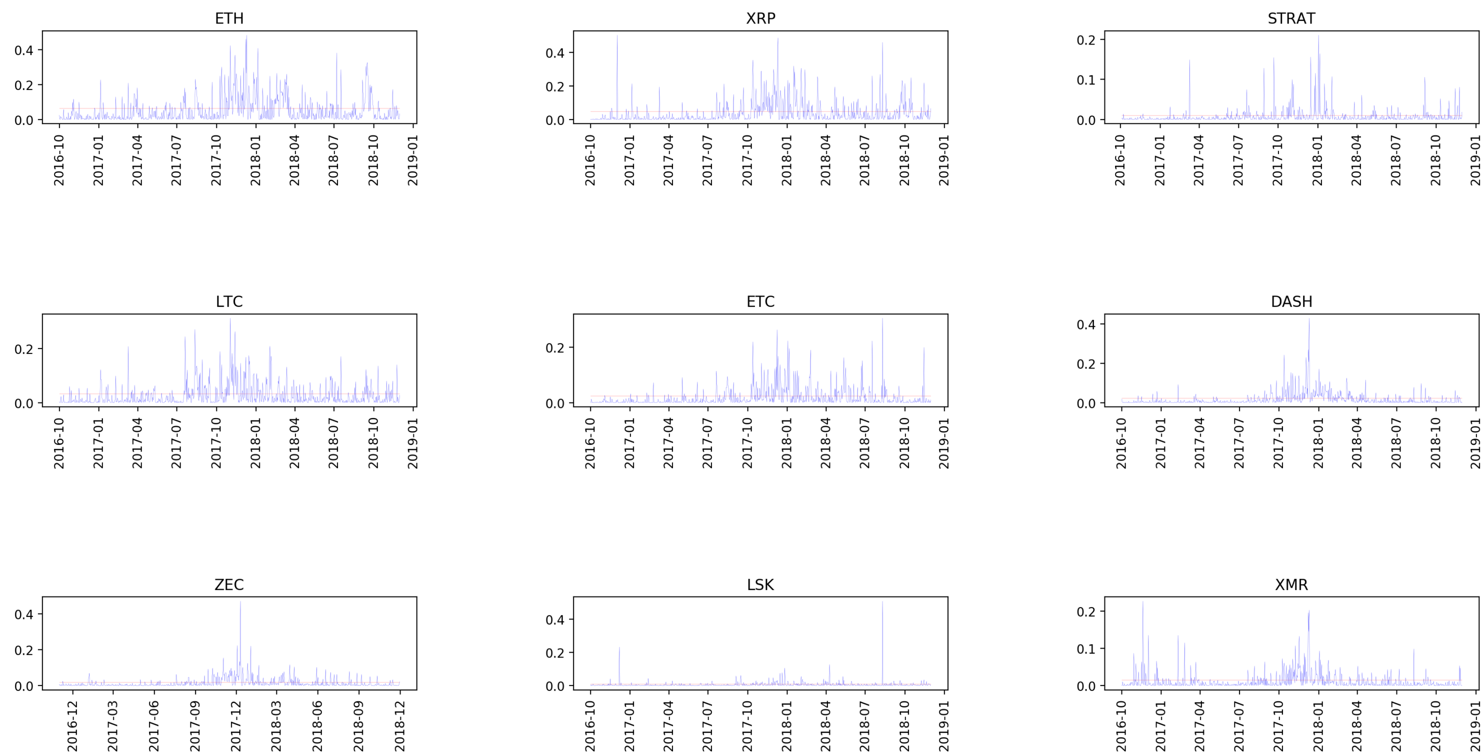

Figure 13: Systematic Risk Ratio (SRR) for Cryptocurrencies

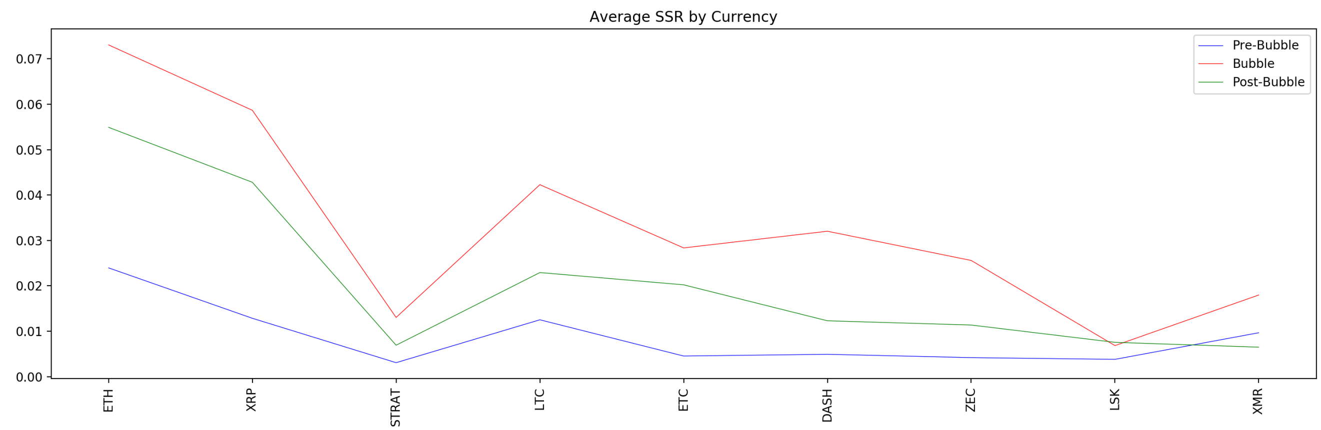

Figure 14: The Average SSR across the Cryptocurrencies

et al. (2019) study to calculate Systematic Risk Ratio (SRR) for cryptocurrency i as :

$$SSR_{i,t} = \frac{R\beta_{i,t}^2 \, RV_{BTC,t}}{RV_{i,t}} \tag{11}$$

Based on the definition of SRR, this ratio gives the fraction of cryptocurrency i's variance explained by Bitcoin's variance. By using intraday high frequency data, we calculate the daily systematic risk ratio for each cryptocurrency throughout the sample period. Figure 13 plots the SSR for each cryptocurrency and the red line is the upper bound of the 99% confidence interval of SSR. There is clear evidence that the Bitcoin variance is a powerful way to explain the cryptocurrency variance during the bubble period. In fact, the SSR reaches its highest level near the peak of the Bitcoin bubble (December 2017) for all cryptos except LSK. This pattern matches the beta plots, which show more negative significant beta clustered during the Bitcoin bubble period. Figure 14 plots the average SSR across the cryptocurrencies. It is clear that, while the fraction of cryptocurrency variance explained by Bitcoin variance is greater during the bubble period and after the bubble burst, the explanatory power of the Bitcoin variance remains elevated compared to the pre-bubble period. Therefore, we assert that:

Fact 9: *The fraction of variance of cryptocurrency explained by the Bitcoin variance is high during the bubble period, and the explained fraction remains at an elevated level in the post-bubble period.*

7 Conclusions

In this study, we have presented a set of stylized facts on cryptocurrency returns and volatility. Specifically, from our analysis of high-frequency tick data on the most liquid nine cryptocurrencies from October 2016 to November 2018, we assert the following:

Fact 1: *Daily realized cryptocurrency volatility has high persistence.*
Fact 2: *The distribution of the logarithm of realized volatility of cryptocurrencies is close to normal.*
Fact 3: *The factor structure in daily cryptocurrency volatility is stronger than the factor structure in returns.*
Fact 4: *Economic and financial factors do not have strong explanatory power on the common factors of cryptocurrency return and volatility and there is a weak inverse relationship between cryptocurrency risk and macroeconomic indices.*

Fact 5: *Bitcoin can be considered for most cryptocurrencies as a fundamental factor able to explain a small proportion of the variations in return and volatility.*
Fact 6: *The Factor Structure model is more powerful in explaining variation in returns and volatilities during the Bitcoin bubble period and this explanatory power persists - and, for volatilities actually increases further - after the Bitcoin bubble burst.*
Fact 7: *There is heterogeneity in the relationship between Bitcoin and other cryptocurrencies for both returns and volatility after the Bitcoin pricing bubble burst.*
Fact 8: *Cryptocurrency betas with Bitcoin were negative before the Bitcoin bubble burst but became positive after the bubble burst.*
Fact 9: *The fraction of variance of cryptocurrency explained by the Bitcoin variance is high during the bubble period, and the explained fraction remains at an elevated level in the post-bubble period.*

Our study uncovers the properties of cryptocurrency and constructs a factor structure model. The cryptocurrencies are strongly explained by their own common factors but not by the fundamental economic factors used in most economics and finance studies. Taking into consideration Bitcoin as a fundamental factor, the nature of the relationship between Bitcoin and other cryptocurrencies shifted in terms of both return and volatility after the Bitcoin bubble burst. The strong common components of volatility across the major cryptocurrencies need to be considered as part of risk management when making investment decisions in cryptocurrency.

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Appendix

A Bitcoin Bubble Dating Calculation

We mainly follow the study by Phillips and Yu (2011) and Phillips et al. (2011) to dating the timeline of Bitcoin bubble during the irrational fanaticism in cryptocurrency market from April 2007 to February 2018. We first run recursive least square regression and estimate the autoregressive specification for Bitcoin price:

$$P_t = \mu + \delta P_{t-1} + \varepsilon_t \quad \varepsilon_t \sim i.i.d.\ (0, \sigma^2) \tag{12}$$

The independent and identically distributed ($i.i.d.$) assumption can also be relaxed to serially dependent errors. The null hypothesis is $H_0 : \delta = 1$ and the right-tailed alternative hypothesis is $H_1 : \delta > 1$ which indicates mildly explosive behavior in the process of Bitcoin price. We initialize our first recursion with 140 observations ($\tau_0 = nr_0$, which $r \in (0,1]$ is a ratio of partitions to entire sample size n). The corresponding coefficient test statistics and Dickey-Fuller t statistics by DF_r^t, namely

$\mathrm{DF}_r^t := \left(\frac{\sum_{j-1}^{\tau} \tilde{X}_{j-1}^2}{\hat{\sigma}_\tau^2} \right) (\hat{\delta}_\tau - 1)(13)$

The successive observations in the subsequent regressions after the first initialization is $\tau = \lfloor nr \rfloor$. $\hat{\sigma}_\tau^2$ is the corresponding estimate of σ^2. $\tilde{X}_{j-1} = X_{j-1} - \frac{\sum_{j=1}^{\tau} X_{j-1}}{\tau}$. The critical value we use to compare the statistical value of Dickey-Fuller test is $cv_{\beta_n}^{df} = -0.08 + \ln(\lfloor nr \rfloor)/C$. Without loss of generality, we choose $C = 5$ to give a conservative test as Phillips and Yu (2011) suggests.

In Phillips and Yu (2011) study, they define the origination of the bubble by estimate $\hat{\tau}_e = \lfloor n\hat{r}_e \rfloor$ as flowing :

$$\hat{r}_e = \inf_{s \geq r_0} \{ s : DF_s^t > cv_{\beta_n}^{df} \} \tag{14}$$

and the collapse of bubble by $\hat{\tau}_f = \lfloor n\hat{r}_f \rfloor$ as following :

$$\hat{r}_f = \inf_{s \geq \hat{r}_e + \gamma \ln(n)/n} \{ s : DF_s^t < cv_{\beta_n}^{df} \} \tag{15}$$

However, the test statistics often dropped below or rose above the relevant critical values between these dates (see Figure 3.11). In real time, dating the bubble would have been difficult wholly based on this approach. But our interest is in historically dating the bubble solely to split our sample into the pre-bubble, bubble and post-bubble periods. We define to find the bubble collapse date as days after the collapse date is consistently showing no mildly explosive behavior in Bitcoin price process.

Finally, we conduct initialization to improve the dating process based on the procedure by Phillips et al. (2011) study. The Bitcoin bubble we find is from May 24, 2017, and the bubble collapse on January 28, 2018.

For full detailed information, we refer to review the comprehensive studies from Phillips and Yu (2011) and Phillips et al. (2011).

B Additional Figures

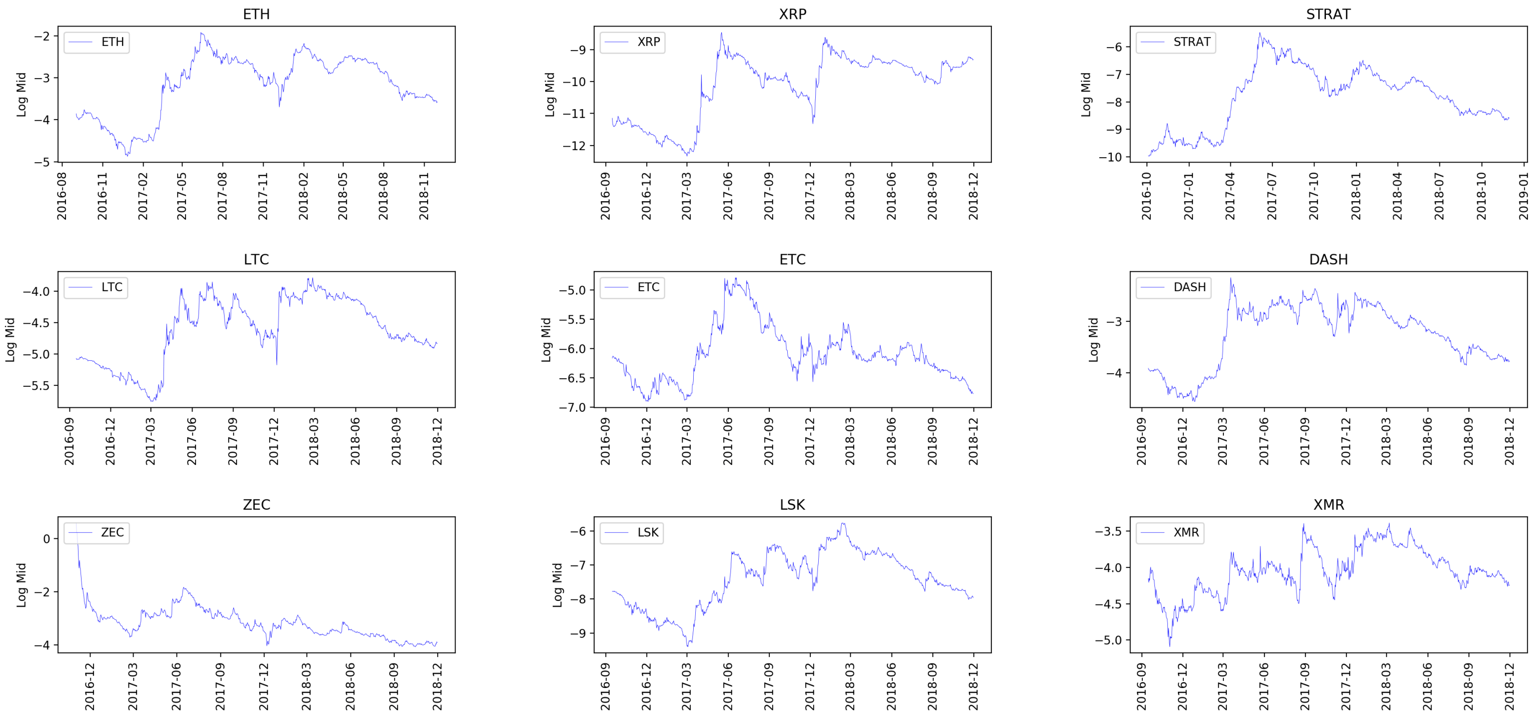

Figure 15: Log Mid Price of Cryptocurrency

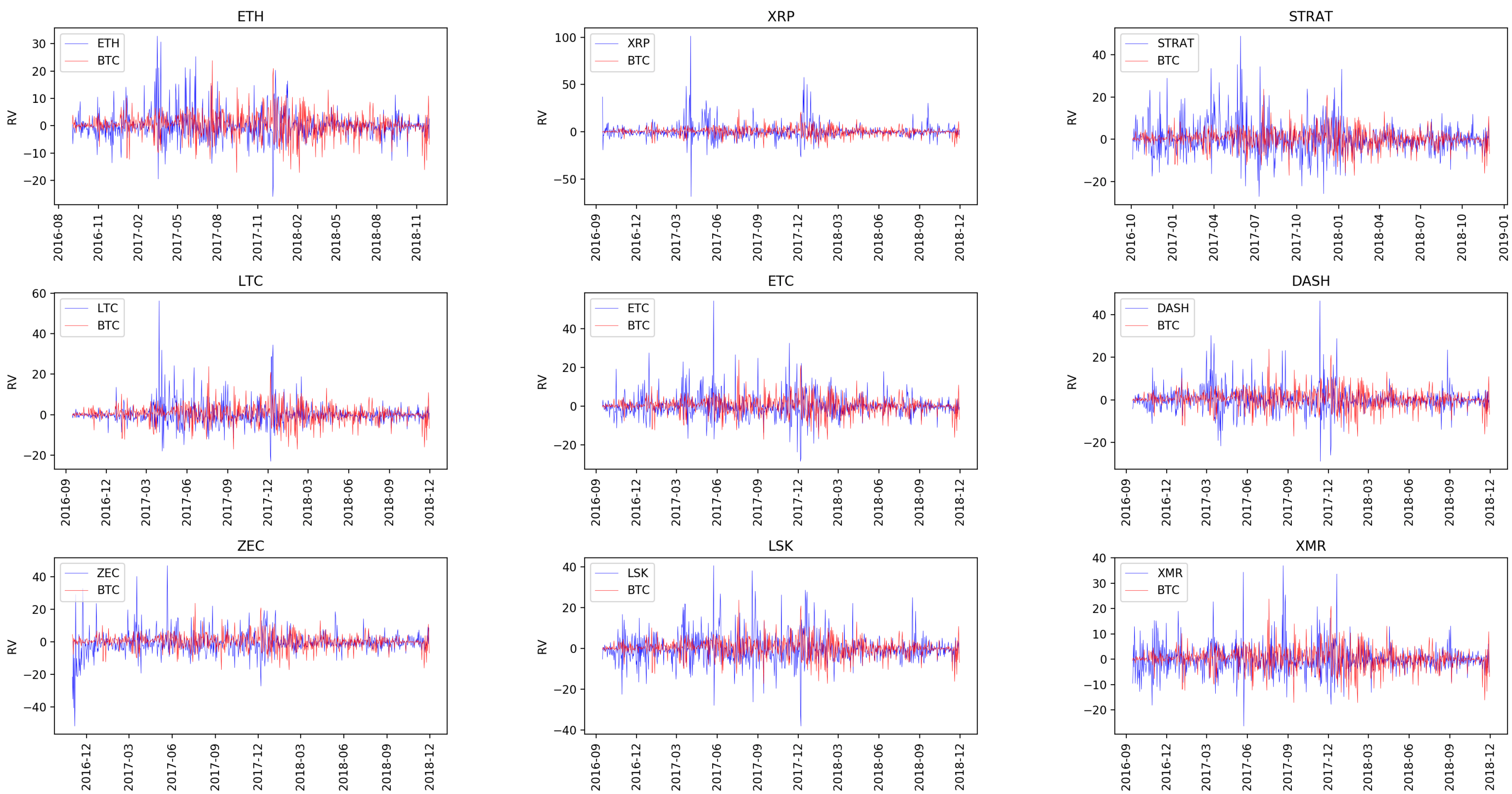

Figure 16: Cryptocurrency Return

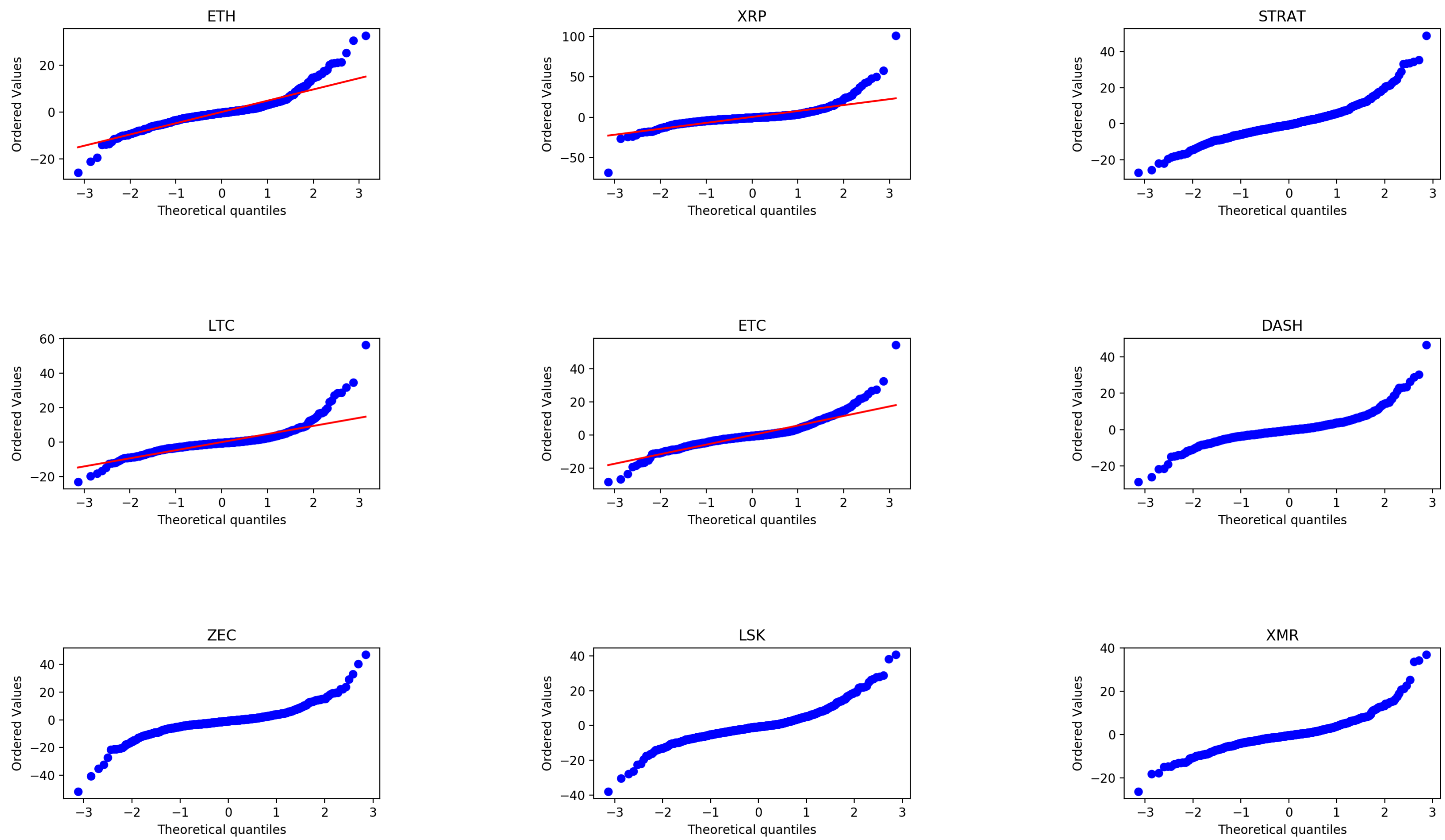

Figure 17: Cryptocurrency Return QQ Plots

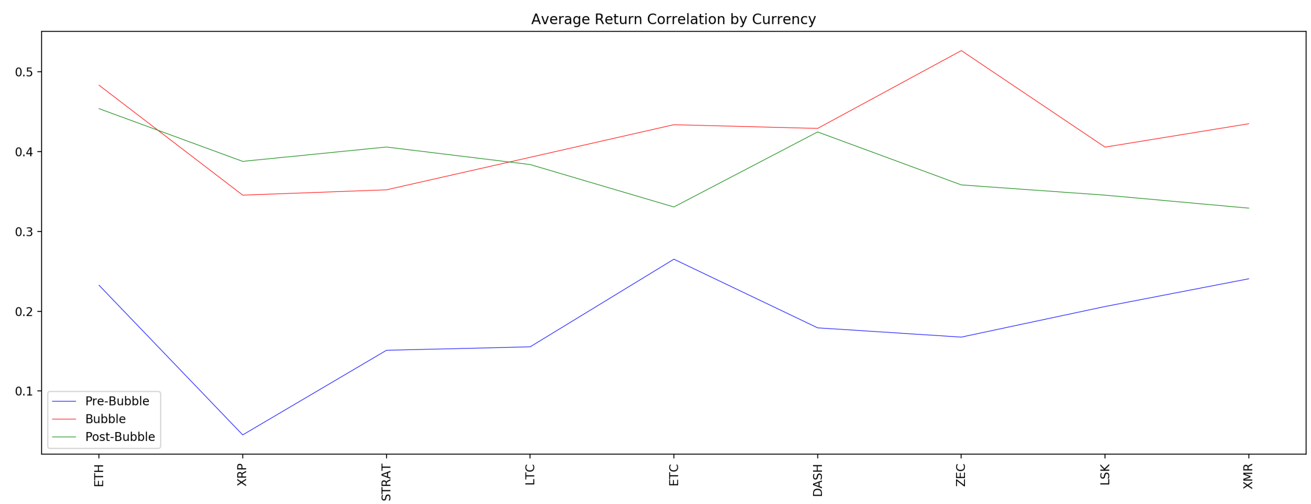

Figure 18: Average Cryptocurrency Return Correlation in Pre-Bubble, Bubble and Post-Bubble

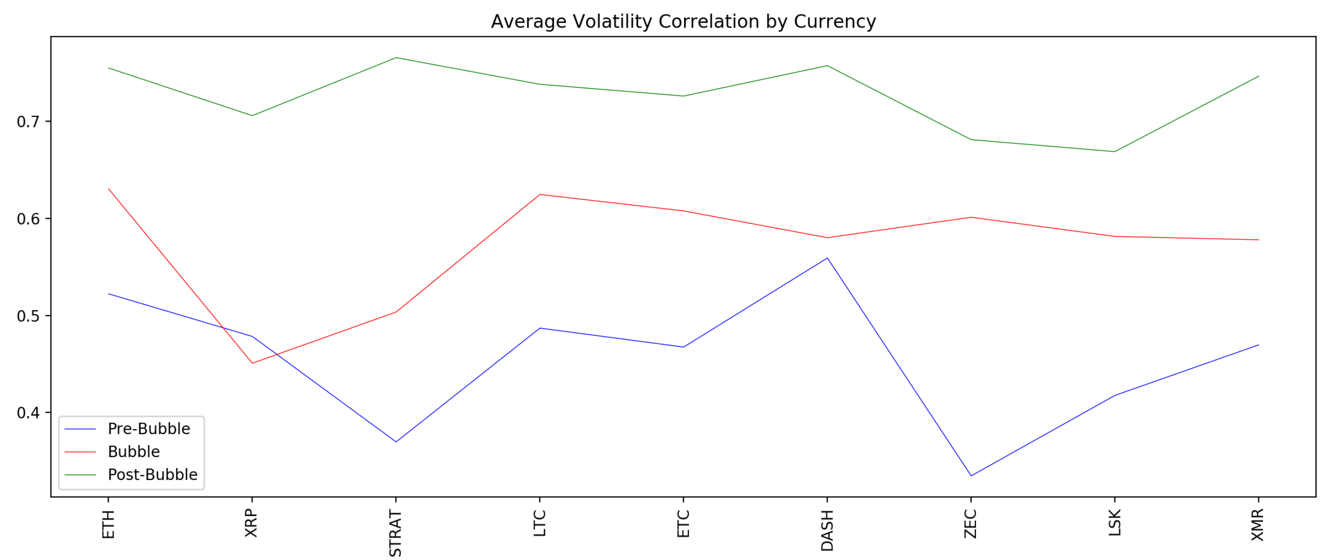

Figure 19: Average Cryptocurrency Volatility Correlation in Pre-Bubble, Bubble and Post-Bubble

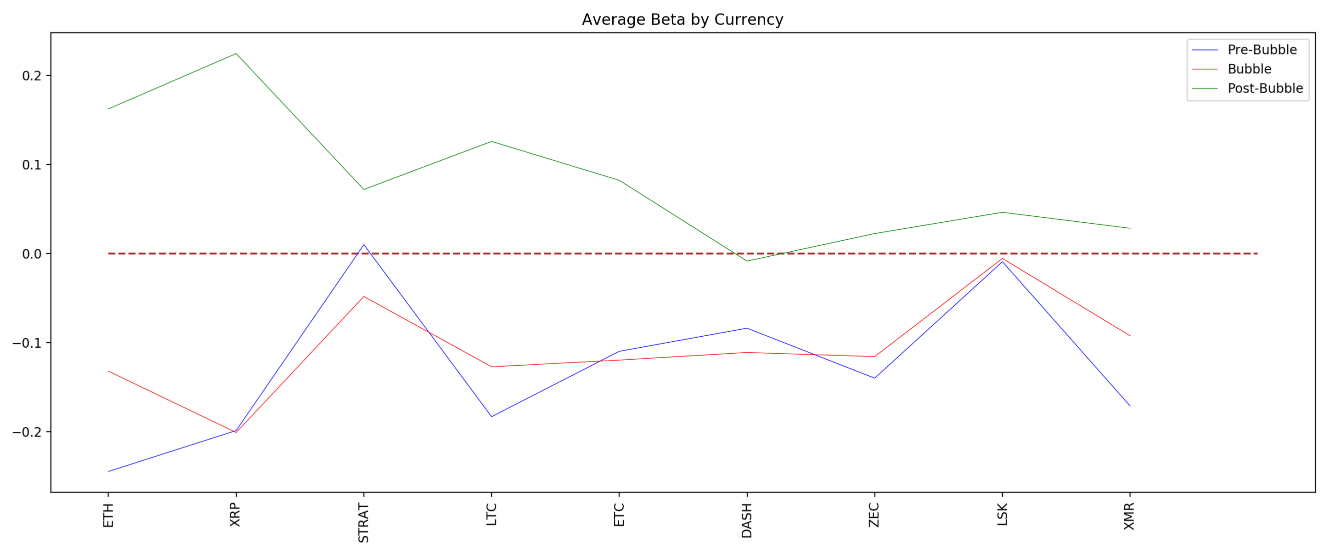

Figure 20: Average Realized Betas in Pre-Bubble, Bubble and Post-Bubble